Loss Uncertainty, Gain Uncertainty, and Expected Stock Returns

Bruno Feunou Bank of Canada Ricardo Lopez Aliouchkin Syracuse University

Roméo Tédongap ESSEC Business School Lai Xu Syracuse University

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Abstract

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Keywords: Cross-section of stocks, out-of-the-money options, variance risk premium JEL Classification: G12

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Abstract

We decompose the quadratic payoff on a stock into its loss and gain components and measure the premia associated with their fluctuations, called the loss and gain quadratic risk premium (QRP) respectively. The loss QRP interprets as the premium paid for downside risk hedging, while the gain QRP reads as the premium received for upside risk compensation. Long-short portfolio strategies based on the loss or gain QRP yield monthly risk-adjusted expected excess returns of up to 2.8%. This cross-sectional predictability survives a battery of robustness checks, and is reinforced among stocks experiencing limits to arbitrage, information asymmetry, and demand for lottery.

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1 Introduction

Economists would agree that the loss and the gain are the main attributes of an investment return. Bernardo and Ledoit (2000) define the loss l and the gain g as magnitudes of the nonpositive and the nonnegative parts of the return r, respectively, that is, $l = \max(-r, 0)$ and $g = \max(r, 0)$. The ex ante perceptions of the potential loss and gain not only determine the attractiveness of an investment opportunity but they are also relevant for its relative valuation. The loss uncertainty characterizes the risk of the actual return being negative, or the uncertainty about the amplitude of the loss. Similarly, the gain uncertainty characterizes the potential of the actual return being positive, or the uncertainty about the size of the gain.

In this paper, we first provide measures of the premia associated with fluctuations in the loss uncertainty and the gain uncertainty, called the loss quadratic risk premium (QRP) and the gain QRP, respectively. Our empirical measurement and estimation of the loss and gain QRPs are consistent with a premium definition as the difference between the risk-neutral and physical expectations of the same quantity. More precisely, we define the loss QRP as the risk-neutral minus physical expectation of quadratic loss, i.e., $\text{QRP}^l \equiv \mathbb{E}^{\mathbb{Q}} [l^2] - \mathbb{E} [l^2]$, so that the loss QRP is positive for investors who are typically averse to fluctuating loss uncertainty. Risk averse investors thus pay the loss QRP to hedge extreme losses in bad times. To the contrary, we define the gain QRP as the physical minus risk-neutral expectation of quadratic gain, i.e., $\text{QRP}^g \equiv \mathbb{E} [g^2] - \mathbb{E}^{\mathbb{Q}} [g^2]$, so that the gain QRP is positive for investors who are typically averse to fluctuating gain uncertainty. Risk averse investors thus receive the gain QRP to compensate for weak upside potential in bad times.

Next, we argue that an asset's premium must reflect its loss QRP and gain QRP. Our reasoning is as follows. An asset with larger loss QRP is unattractive because a higher loss QRP reflects more severe downside risk in bad times. Likewise, an asset with larger gain QRP is unattractive because a higher gain QRP means weaker gain potential in bad times. Since investors are sensitive to fluctuations in loss (gain) uncertainty, they would require a higher premium for holding assets with higher loss (gain) QRP. Those assets will in turn pay higher returns on average.

We empirically explore our cross-sectional predictions using stock and option data for the U.S. from January 1996 to December 2015. To measure risk-neutral expectations, we exploit results from Bakshi and Madan (2000) and Bakshi, Kapadia, and Madan (2003) to prove that the riskneutral expected quadratic loss (gain) can be recovered from the market prices of out-of-the-money European put (call) options. Option data are used to implement these formulas. A conditional log normality of returns is assumed to derive analytical formulas for the physical expectations of quadratic gain and quadratic loss. A variant of the heterogenous autoregressive model of the realized volatility (HAR-RV) of Corsi (2009) is used to estimate the conditional variance and the same information set is used to estimate the conditional mean of log returns. Stock data are used to implement physical expectations formulas. Our measures for the loss and gain QRPs are the appropriate difference between the corresponding risk-neutral and physical expectations.

In our main cross-sectional tests, we use portfolio sorts based on each firm's QRP components (i.e., the loss and gain QRPs), controlling for exposures to frequently investigated market factors and other firm characteristics. Across firms, we find a wide dispersion in QRP components which generates cross-sectional variations in asset premia. We find strong evidence that the QRP components are positively related to expected excess returns in the cross-section. Specifically, simultaneously going long a portfolio of firms with high loss QRP and short a portfolio of firms with low loss QRP yields a monthly expected excess return of 2.79%, risk-adjusted using the five-factor model of Fama and French (2015). Likewise, we also find that the gain QRP has a strong positive and significant relation with monthly expected stock returns. The long-short portfolio has a five-factor alpha of 2.78% per month.¹ Since the two QRP components have similar effects in the cross-section, and QRP is by definition the difference between its two components (we also refer to QRP as the net QRP), this potentially explains why we find no evidence of a relation between (the net) QRP and monthly expected stock returns. Thus, decomposing the QRP into its loss and gain components is clearly very informative.

We run Fama and MacBeth (1973) cross-sectional regressions with individual stocks as test assets to estimate risk prices associated with the QRP components. Cross-sectional regression results confirm that the QRP components provide significant explanatory power for the variation of monthly expected stock returns beyond traditional asset pricing risk factors and firm characteristics. Our estimates suggest that, everything else being equal, the QRP components are economically

¹At the first glance these spreads may seem high. However, once we control for trading costs and microstructure effects that are not tradeable, these spreads decrease and are of similar size to previous literature on other anomalies such as the asset growth anomaly (Cooper, Gulen, and Schill, 2008), or the idiosyncratic volatility puzzle (Ang, Hodrick, Xing, and Zhang, 2006).

important that a one standard deviation increase is associated with a rise in monthly expected excess returns between 1.7% and 2.4% in the cross-section.

Our paper mostly contributes to the literature on the cross-sectional implications of downside risk (e.g., Ang, Chen, and Xing, 2006; Lettau, Maggiori, and Weber, 2014; Farago and Tédongap, 2018). Our measure for downside risk, the loss QRP corresponds to the specific cost to insure against undesirable fluctuations in a firm's loss uncertainty. Since we use the quadratic payoff rather than the payoff itself, the loss QRP partly represents a firm's return squared exposure or squared beta relative to market-wide factors, but also partly represents firm characteristics related to the idiosyncratic variance or the jump variation of the returns. Empirical tests and evidence in Daniel and Titman (1997, 2012) support our approach of measuring the downside risk through a firm's specific characteristic rather than its factor exposure. Thus, our paper is related to Xing, Zhang, and Zhao (2010) and Yan (2011) who show that the firm-level implied volatility smirk (an optionbased measure of downside risk) has a strong predictive power for expected stock returns. It also relates to Bollerslev, Li, and Zhao (forthcoming) who find that the signed jump variation (defined as the standardized difference between the gain and loss realized variances) is significantly related to expected stock returns, and Huang and Li (2019) who investigate the risk-neutral counterpart of the signed jump variation. In our empirical analyses, we control for the implied volatility smirk and the signed jump variation, as well as multivariate exposures to the generalized disappointment aversion (GDA) factors of Farago and Tédongap (2018), and find that the loss and gain QRPs still have significant positive relationships with expected stock returns in our sample. Besides that, our result regarding the gain QRP shows that the upside risk is significantly and robustly priced even after controlled for the downside risk. Since there is little evidence in the literature about the pricing of the upside risk, our findings on the gain QRP constitute an important new contribution.

A popular measure of the premium for bearing fluctuating uncertainty is the variance risk premium (VRP). In previous literature, VRP has been examined for the aggregate stock market's time series predictability (e.g., Bollerslev, Tauchen, and Zhou, 2009, Bollerslev, Marrone, Xu, and Zhou, 2014, Feunou, Jahan-Parvar, and Okou, 2018 and Kilic and Shaliastovich, 2019) as well as for the cross-sectional predictability (e.g., Han and Zhou, 2011). However, there is a lack of coherency in the literature as to how to accurately estimate and measure VRP and its loss and gain components. While the physical expectation of realized variance is consistently estimated using an appropriate time series forecasting model, its risk-neutral expectation is, in general, estimated via a Bakshi, Kapadia, and Madan (2003)-like formula which corresponds to the risk-neutral expectation of quadratic payoff. As a result, the estimated VRP in previous studies does not conform with a premium definition. This measure is biased unless the quadratic payoff and the realized variance are equal. We illustrate the significance of this bias by using the S&P 500 daily and intra-daily return data. Furthermore, we show that the loss and gain components of the quadratic payoff are significantly different from their counterparts for the realized variance (the so-called semi-variances). Other types of bias related to the measurement of risk-neutral second moments of returns and in connection with the options-implied volatility index (VIX) are discussed by Andersen, Bondarenko, and Gonzalez-Perez (2015) and Martin (2017). In this paper, by focusing on QRP and its components, we can maintain the premium definition and be free from this significant bias between the realized variance and the quadratic payoff.

Our results also appear useful for understanding important asset pricing anomalies put forward in the recent literature. Stambaugh, Yu, and Yuan (2015) find that idiosyncratic volatility is negatively priced among overpriced stocks, and this cross-sectional predictability is the highest among overpriced stocks that are also difficult to short. Similarly, we find that idiosyncratic volatility is significantly negatively priced only among stocks with low loss QRP, and within this group, its cross-sectional predictability is the highest among stocks with low gain QRP. Stocks with low loss QRP are preferred by the investors because they have small downside risk in bad times. Thus investors' extra demand leads to the relative overpricing of these stocks. Further, among stocks with low loss QRP, those with low gain QRP have large upside potentials in bad times, thus are more desirable and shorting them may be risky and very costly. Taken together, these results corroborate and extend, using our downside and upside risk measures, the arbitrage asymmetry and arbitrage risk explanations of the idiosyncratic volatility puzzle in Stambaugh, Yu, and Yuan (2015) for a large sample of optionable stocks.

Our results finally evidence that cross-sectional predictability of the loss and gain QRPs is not uniform across all categories of stocks, i.e., it is significantly stronger for certain types of stocks relative to others. This suggests that a particular characteristic may be essential for understanding why certain stocks are more predictable by the QRP components in the cross-section relative to others. In particular, we find that the cross-sectional predictability of the loss and gain QRPs is the strongest among firms for which illiquidity may prevent rational arbitrageurs from exploiting existing arbitrage opportunities. Likewise, we find that as the diffusion of firm-specific information increases, as proxied by the number of analysts covering the stock, the predictability of both the loss and gain QRPs decreases. These results suggest that the predictability of the loss and gain QRPs is in part driven by limits to arbitrage and information asymmetry. We also find evidence that the cross-sectional predictability of the gain QRP is in part driven by the demand for lottery, as proxied by the MAX measure of Bali, Cakici, and Whitelaw (2011).

The rest of the paper is organized as follows. Section 2 introduces and motivates QRP and discusses its relation with VRP. Section 3 discusses the methodology used to estimate individual firm QRP components. Section 4 discusses the data and presents descriptive statistics of the key measures. In Section 5, we investigate the cross-sectional relationship between QRP components and expected stock returns. Section 6 discusses possible ways for explaining and understanding our findings. Section 7 concludes. An Internet Appendix available on the authors' webpages contains details on analytical proofs, data sources and the measurement of factor exposures and firm characteristics, as well as results and illustrations that are omitted for brevity.

2 Theory and Motivation

In this section we formally define QRP and its gain and loss components, which we then compare to VRP and its components. In the case of a monthly horizon, the quadratic payoff is the squared log return over a month, while the realized variance is the sum of squared daily (or higher frequency) log returns within a month. Although both are valid nonparametric measures of stock return uncertainty, the quadratic payoff may be very different from the realized variance and we formally illustrate their difference. This difference is more pronounced between the loss and gain components of the quadratic payoff (called quadratic loss and gain, respectively) and their counterparts for the realized variance (called semi-variances). Consequently, the realized semi-variances cannot be substituted by the quadratic loss and quadratic gain when measuring the VRP components.

2.1 Quadratic Risk Premium: Decomposition and Interpretation

We introduce QRP, the difference between the risk-neutral and physical expectations of quadratic payoff (squared log return). Formally, denote $r_{t-1,t}$ the monthly realized (log) return from end of month t-1 to end of month t. The quadratic payoff is simply $r_{t-1,t}^2$, and is a measure of fluctuating uncertainty over the monthly period. Risk-averse investors dislike fluctuating uncertainty because large fluctuations may lead to high uncertainty levels, which in turn may result in losses.

The QRP can be interpreted as the net outflow of a risk-averse investor in a quadratic swap market. In theory, an investor who dislikes fluctuating uncertainty would be willing to swap it for a fixed amount. We can define the quadratic strike as the fixed amount an investor would request against fluctuating quadratic payoff. To the best of our knowledge, quadratic swap markets do not exist. Thus being able to compute the quadratic strike of an asset from available data provides an assessment of the insurance cost for hedging its fluctuating uncertainty. On the other hand, since measuring uncertainty through the realized variance is common in the literature, we can also consider a variance swap market. In this market, risk-averse investors can swap the variance for a fixed amount, called the variance strike, which is directly observable for a minority of stocks that have functioning variance swap markets. For the majority of stocks, however, the variance strike has to be estimated. We choose to use the quadratic payoff to measure uncertainty and QRP as the net insurance cost because the estimation of the quadratic strike is feasible using option data, while the variance strike is not (see Section 3.1 for details).

The QRP is positive on average because investors are typically risk-averse and dislike fluctuating uncertainty. Risk-averse investors swapping the strike against the fluctuating uncertainty will be better off if the uncertainty level turns out to be largely above the strike paid. For the privilege of savoring this outcome in hard times when the marginal utility is high, investors would be willing to pay an insurance cost. The strike minus the (physical) expected uncertainty level would be positive, thus representing the positive QRP. Since a swap has zero net market value at inception, the no-arbitrage condition dictates that the strike is equal to the risk-neutral expected uncertainty level. We formally define QRP as follows:

$$QRP_{t} \equiv \mathbb{E}_{t}^{\mathbb{Q}} \left[r_{t,t+1}^{2} \right] - \mathbb{E}_{t} \left[r_{t,t+1}^{2} \right]$$

$$= Cov_{t} \left(M_{t,t+1}, r_{t,t+1}^{2} \right), \qquad (1)$$

where $\mathbb{E}_t [\cdot]$ denotes the time-*t* physical conditional expectation operator, $\mathbb{E}_t^{\mathbb{Q}} [\cdot]$ denotes the time*t* conditional expectation operator under some risk-neutral measure \mathbb{Q} , $M_{t,t+1}$ is the state price density used to price assets between time *t* and time t + 1, and $Cov_t(\cdot, \cdot)$ is the time-*t* physical conditional covariance operator.

Equation (1) shows that the QRP is fully characterized by the systematic risk of the quadratic payoff. Notice however that the QRP is not free from the idiosyncratic volatility as usually understood. Indeed, the firm returns can be written as $r_{t,t+1} = \beta_t (M_{t,t+1}) + \varepsilon_{t+1}$ where $\beta_t (M_{t,t+1}) = \mathbb{E}_t [r_{t,t+1} | M_{t,t+1}]$ is the systematic component of the returns, and ε_{t+1} is the idiosyncratic component of the returns with $\mathbb{E}_t [\varepsilon_{t+1} | M_{t,t+1}] = 0$. Let $\vartheta_t (M_{t,t+1})$ denote the variance of ε_{t+1} conditional on $M_{t,t+1}$, that is, $\mathbb{E}_t [\varepsilon_{t+1}^2 | M_{t,t+1}] = \vartheta_t (M_{t,t+1})$, i.e., $\vartheta_t (M_{t,t+1})$ is the idiosyncratic variance of the returns. It follows that QRP $_t = Cov_t (M_{t,t+1}, \beta_t^2 (M_{t,t+1})) + Cov_t (M_{t,t+1}, \vartheta_t (M_{t,t+1}))$. This shows that, even though the QRP is free from idiosyncratic risk of the quadratic payoff, it is not free from the idiosyncratic volatility of the return (the payoff itself). Instead, the QRP is partly characterized by the idiosyncratic variance of the returns and related firm characteristics.

We now decompose the asset return r and the quadratic payoff r^2 into a gain and a loss component as follows:

$$r = g - l$$
 and $r^2 = g^2 + l^2$, where $g = \max(r, 0)$ and $l = \max(-r, 0)$, (2)

where g and l represent the gain and the loss, respectively. In this decomposition, the gain and the loss are nonnegative amounts flowing in and out of the investor's wealth, and they represent the magnitudes of the nonnegative and nonpositive parts of the asset payoff, respectively. Since the positive gain and the positive loss cannot occur simultaneously, we have that $g \cdot l = 0$. This gainloss decomposition of an asset's payoff is exploited as an asset pricing approach by Bernardo and Ledoit (2000). Since a typical investor prefers a large gain g and a small loss l, the gain uncertainty (measured by the quadratic gain g^2) thus appears as a good uncertainty while the loss uncertainty (measured by the quadratic loss l^2) is a bad uncertainty. These views are consistent with the literature documenting that good and bad variances are not equally undesirable by investors.²

Just as the return uncertainty fluctuates, its two components, the loss uncertainty and the gain uncertainty, do too. Investors are typically averse to fluctuating loss uncertainty because large loss fluctuations may lead to strong loss uncertainty levels and extreme losses. They would typically be willing to swap this fluctuating quadratic loss against a strike higher than the expected quadratic loss — pay a positive loss QRP — to enjoy being better off in bad times when the quadratic loss significantly outperforms the strike. Likewise, risk-averse investors dislike fluctuating gain uncertainty because large fluctuations may lead to weak uncertainty levels and poor gain potential. Therefore, investors would typically be willing to swap fluctuating quadratic gain against a strike lower than the expected quadratic gain — require a positive gain QRP — to endure being worse off in bad times when the quadratic gain significantly falls below the strike.

Consistent with these views, we define the loss QRP and the gain QRP as follows:

so that they are positive if uncertainty levels tend to move adversely in hard times when the average investor's marginal utility $M_{t,t+1}$ is high.³ Thus, using the gain-loss decomposition of the quadratic payoff in equation (2), the (net) QRP in equation (1) may be written as:

$$QRP_t = QRP_t^l - QRP_t^g.$$
(4)

Equation (4) shows that the (net) QRP represents the net cost of insuring fluctuations in loss uncertainty, that is the premium paid for the insurance against fluctuations in loss uncertainty net

²For example, Markowitz (1959) advocates the downside semi-variance (i.e, the bad variance) as a measure of a stock's downside risk, instead of the total variance, because the latter also accounts for the upside semi-variance (i.e, the good variance), which measures the gain potential of a stock. More recently, Feunou, Jahan-Parvar, and Tédongap (2013), Bekaert, Engstrom, and Ermolov (2015), and Segal, Shaliastovich, and Yaron (2015) find that expected excess returns are positively (negatively) related to the bad (good) variance. This suggests that investors are averse to the increases in the bad variance yet they also desire increases in the good variance.

³The pricing kernel $M_{t,t+1}$ is equal to the growth in the marginal value of the investor's wealth (Cochrane, 2005).

of the premium earned to compensate for the fluctuations in gain uncertainty.⁴

2.2 The Cross-Section of Quadratic Risk Premium and Expected Stock Returns

We can measure QRP at the aggregated market level or the disaggregated firm-level. For either the market or firm-level, by definition, the QRP has a premium interpretation as evident in equation (1). Although the state price density $M_{t,t+1}$ in this equation is free from the linear factor-based specification, we can still use the linear framework to illustrate the intuition. In the linear case where $M_{t,t+1}$ is assumed to be a linear combination of various systematic factors, equation (1) would relate QRP to the weighted sum of the covariances between the stock's quadratic payoff and each of these systematic factors.⁵ This suggests that, at the firm-level, the idiosyncratic component of the quadratic payoff that is orthogonal to the systematic factors is not accounted for by its QRP. In light of equation (3), this is also the case for the loss and gain QRPs which means that our measures of downside and upside risk are free of the idiosyncratic risk in the quadratic loss and gain, respectively, but are still determined by the idiosyncratic volatility of returns. Also, since our risk measures are not exposures of excess returns themselves (but rather the quadratic payoff) onto systematic factors, nor are they obtained as betas through times series regressions, they can be viewed as characteristics similar to size, book-to-market, momentum, or idiosyncratic volatility.

To provide the theoretical predictions of the cross-sectional relation between the individual stock QRP components and expected excess returns, we consider the risk-reward point of view. Since investors dislike assets with higher downside risk, they should require higher expected returns for holding those assets. The downside risk of an asset measured by its fluctuating loss uncertainty is undesirable as large fluctuations may lead to strong uncertainty levels and extreme losses. The

⁴In a long-run risk model, Held, Kapraun, Omachel, and Thimme (2018) compute the two components of QRP (which they refer to as the premia on second semi-moments) of the aggregate stock market and confirm that the loss and gain QRPs as defined in equation (3) are positive. This illustrates that, for an asset for which the uncertainty moves together with the average investor's marginal utility, the cost of insuring against fluctuations in loss uncertainty exceeds the compensation for being exposed to fluctuations in gain uncertainty, and QRP measures by how much.

⁵With a set of identified factors, equation (1) can be formally tested to determine whether the cross-sectional differences in QRP across stocks are explained by the cross-sectional differences in exposures of the quadratic payoff on the systematic factors. González-Urteaga and Rubio (2016) address this issue in the case of the variance risk premium by using selective groups of systematic factors including the market return together with the squared market return, and the market variance risk premium together with the default premium (calculated as the difference between Moody's yield on Baa corporate bonds and the ten-year Treasury bond yield). Their findings suggest that the market variance risk premium and the default premium are key factors explaining the average variance risk premium across stock portfolios.

positive loss QRP paid by investors is to insure against this downside risk in bad times. Since this insurance premium increases as the degree of damage increases in bad times, assets with high loss QRP must command higher expected excess returns in the cross-section.

A similar reasoning applies to the gain QRP. Since investors dislike assets with higher upside risk, they should require a higher expected return for holding them. An asset's upside risk measured by its fluctuating gain uncertainty is undesirable because large fluctuations may lead to weak uncertainty levels and poor gains. The positive gain QRP is required by investors to compensate for this low upside potential in bad times. Since this compensation increases as the degree of shrink in gains increases in bad times, assets with high gain QRP must command higher expected excess returns in the cross-section.

In Section 5, we present the empirical results of the cross-sectional relation between individual stock loss and gain QRP and expected excess returns.

2.3 Relation with the Variance Risk Premium

We next discuss the relation between QRP and VRP. Both QRP and VRP share the premium definition but they regard different measures of uncertainty: the quadratic payoff versus the realized variance. Therefore, the difference between QRP and VRP hinges on the difference between the quadratic payoff and the realized variance. For a given stock, we observe returns at regular highfrequency time intervals of length δ . The monthly realized return $r_{t-1,t}$ and the monthly realized variance $RV_{t-1,t}$ are defined by aggregating $r_{t-1+j\delta}$ and $r_{t-1+j\delta}^2$, respectively:

$$r_{t-1,t} = \sum_{j=1}^{1/\delta} r_{t-1+j\delta}$$
 and $\operatorname{RV}_{t-1,t} = \sum_{j=1}^{1/\delta} r_{t-1+j\delta}^2$, (5)

where $1/\delta$ is the number of high-frequency returns in a monthly period, e.g., $\delta = 1/21$ for daily returns and $r_{t-1+j/21}$ denotes the *j*th high-frequency return of the monthly period starting from day t-1 and ending on day t. The quadratic payoff and the realized variance are related as follows:

$$r_{t-1,t}^{2} = \mathrm{RV}_{t-1,t} + 2\mathrm{RA}_{t-1,t}, \text{ where } \mathrm{RA}_{t-1,t} = \sum_{i=1}^{1/\delta - i} \sum_{j=1}^{1/\delta - i} r_{t-1+j\delta} r_{t-1+j\delta+i\delta},$$
(6)

and $RA_{t-1,t}$ is the realized autocovariance.

The realized variance is a measure of fluctuating uncertainty based on higher-frequency returns, while the quadratic payoff is a measure of fluctuating uncertainty based on lower-frequency returns. Equation (6) shows that the quadratic payoff is approximately equal to the realized variance if and only if the realized autocovariance is negligible. To examine whether this is the case, we take daily S&P 500 index return data as an example. In Panel A of Figure 1, we plot the monthly realized autocovariance of the index in squared percentage unit from January 1996 to December 2015. This figure shows that the realized autocovariance is negative 71.25% of the time with the 95% confidence interval equal to [69.27%, 73.23%], thus the quadratic payoff is frequently smaller than the realized variance. To further prove that the realized autocovariance is non-negligible, we standardize it by computing its ratio relative to the average of the quadratic payoff and the realized variance. In Panel B of Figure 1, we plot the monthly standardized realized autocovariance represents on average about 50.90% (with the 95% confidence interval equal to [49.58%, 52.23%])— a sizeable portion of the uncertainty level.

The realized variance computed from daily returns may contain considerable noise. To nonparametrically correct this bias, prior studies advocate the use of high-frequency intra-day return data. Therefore, we use 5-min intra-day and overnight returns to compute an alternative measure of the realized variance. Results are available in Figure B1 in the Internet Appendix. In summary, the realized autocovariance is negative 67.08% of the time with the 95% confidence interval equal to [61.14%, 73.03%], thus the quadratic payoff is again frequently smaller than the realized variance, but to a slightly lesser degree. We also find that the standardized RA's absolute value averages to 0.50; thus, the realized autocovariance represents on average about 49.99% (with the 95% confidence interval equal to [46.13%, 53.86%]) — again a sizeable portion of the uncertainty level.

To study the difference between QRP and VRP, we adopt the theoretical definition of VRP in Bollerslev, Tauchen, and Zhou (2009) as follows::

$$VRP_{t} \equiv \mathbb{E}_{t}^{\mathbb{Q}} [RV_{t,t+1}] - \mathbb{E}_{t} [RV_{t,t+1}]$$

$$= \sum_{j=1}^{1/\delta} \left(\mathbb{E}_{t}^{\mathbb{Q}} [r_{t-1+j\delta}^{2}] - \mathbb{E}_{t} [r_{t-1+j\delta}^{2}] \right).$$
(7)

In the empirical exercises of the VRP literature, the risk-neutral expectation of quadratic payoff $\mathbb{E}_{t}^{\mathbb{Q}}\left[r_{t,t+1}^{2}\right]$ is often used to proxy for the risk-neutral expected realized variance $\mathbb{E}_{t}^{\mathbb{Q}}\left[\mathrm{RV}_{t,t+1}\right]$. This is, for example, the case in Feunou, Jahan-Parvar, and Okou (2018) and Kilic and Shaliastovich (2019). By doing so, they use an empirical measure of the variance risk premium $\widetilde{\mathrm{VRP}}_{t}$ defined by:

$$\widetilde{\operatorname{VRP}}_{t} = \mathbb{E}_{t}^{\mathbb{Q}} \left[r_{t,t+1}^{2} \right] - \mathbb{E}_{t} \left[RV_{t,t+1} \right]$$

$$= \operatorname{VRP}_{t} + 2\mathbb{E}_{t}^{\mathbb{Q}} \left[RA_{t,t+1} \right].$$
(8)

By definition, $\widetilde{\text{VRP}}_t$ is not a coherent measure of a risk premium (i.e., it is not the difference between the risk-neutral and physical expectations of the same quantity). Instead, $\widetilde{\text{VRP}}_t$ is a biased measure of VRP_t, where the bias equals $2\mathbb{E}_t^{\mathbb{Q}}[RA_{t,t+1}]$. Furthermore, this bias is not necessarily negligible. As shown in panels A and B of Figure 1, $RA_{t,t+1}$ of the S&P 500 index is non-negligible and mostly negative through time. We then cannot reasonably argue that the bias $2\mathbb{E}_t^{\mathbb{Q}}[RA_{t,t+1}]$ or the difference between VRP and $\widetilde{\text{VRP}}_t$ is negligible. While we provide an illustration in the case of the market index, we have strong reasons to believe that this non-negligible bias in the VRP measurement extends to a large number of stocks.

Lastly, we argue that this bias from the realized autocovariance is even more severe when we decompose VRP into its loss and gain components. The gain-loss decomposition of the squared return in equation (2) allows us to write the realized variance as the total of two components: the cumulative sum of squared high-frequency gains and the cumulative sum of squared high-frequency losses, which, similar to the quadratic gain and the quadratic loss can be interpreted as measures of gain uncertainty and loss uncertainty, respectively. These two components of the realized variance are what Barndorff-Nielsen, Kinnebrock, and Shephard (2010) refer to as realized semi-variances, formally defined as:

$$RV_{t-1,t} = RV_{t-1,t}^g + RV_{t-1,t}^l \text{ where } RV_{t-1,t}^g = \sum_{j=1}^{1/\delta} g_{t-1+j\delta}^2 \text{ and } RV_{t-1,t}^l = \sum_{j=1}^{1/\delta} l_{t-1+j\delta}^2.$$
(9)

where $\operatorname{RV}_{t-1,t}^g$ and $\operatorname{RV}_{t-1,t}^l$ are referred to as bad and good variances in the literature (e.g., Patton and Sheppard 2015; Kilic and Shaliastovich 2019; Bollerslev, Li, and Zhao forthcoming). Note that, even if a negligible magnitude of the realized autocovariance made the quadratic payoff $r_{t-1,t}^2$ proxy for the realized variance $RV_{t-1,t}$, the quadratic loss (gain) would not proxy for the loss (gain) realized variance. In fact, the quadratic loss (gain) is a censored variable while the loss (gain) realized variance is not. Therefore, the quadratic loss (gain) is zero 63% (37%) of the time in our S&P 500 index return sample while the loss (gain) realized variance is always positive and can be strongly positive at times, as illustrated in Panel C (D) of Figure 1.

In summary, we show that the quadratic payoff can be very different from the realized variance. This is also partly because the equity risk premium at a lower frequency is non-zero and timevarying. The empirical evidence indeed supports this large wedge between the quadratic payoff and the realized variance at a monthly frequency. We also find that this difference is much more significant between the quadratic loss and gain and their corresponding semi-variances.

3 Measuring the Quadratic Risk Premium

Measuring the QRP amounts to estimating the physical and risk-neutral conditional expectations of quadratic payoff and taking their difference. In this section, we describe our estimation methodology for these two conditional expectations and their loss and gain components. Both the risk-neutral expected quadratic loss and gain are model-free following Bakshi, Kapadia, and Madan (2003). We measure both the physical expected quadratic loss and gain as projections on the space spanned by historical loss and gain realized variances.⁶

3.1 Estimating the Risk-Neutral Conditional Expected Quadratic Payoff

In practice, prior studies estimate the risk-neutral conditional expectation of quadratic payoff directly from a cross-section of option prices. Bakshi, Kapadia, and Madan (2003) provide model-free formulas linking the risk-neutral moments of the stock returns to explicit portfolios of options. These formulas are based on the basic notion, first presented in Bakshi and Madan (2000), that

⁶In theory, these expectations are conditional on the same information set. While asset pricing models imply that both the physical and risk-neutral conditional expectations of uncertainty measures depend on the same processes governing the state of the economy (e.g., Bollerslev, Tauchen, and Zhou, 2009; Drechsler and Yaron, 2011; Bonomo, Garcia, Meddahi, and Tédongap, 2015), this theoretical implication is hard to satisfy. This mismatch of conditioning information in the measurement of the two conditional expectations may explain some differences between theory and practice. For example, the estimates of the QRP as defined in equation (1) may display negative values for the aggregate stock market although theory predicts they should be positive. The same holds for the VRP (see, for example, the plots of the aggregate stock market VRP in Bollerslev, Tauchen, and Zhou, 2009).

any payoff over a time horizon can be spanned by a set of options with different strikes with the same maturity matched with this investment horizon.

We adopt the notation in Bakshi, Kapadia, and Madan (2003), and define $V_t(\tau)$ as the time-tprice of the τ -maturity quadratic payoff on the underlying stock. Bakshi, Kapadia, and Madan (2003) show that $V_t(\tau)$ can be recovered from the market prices of out-of-the-money (OTM) call and put options as follows:

$$V_t(\tau) = \int_{S_t}^{\infty} \frac{1 - \ln\left(K/S_t\right)}{K^2/2} C_t(\tau; K) \, dK + \int_0^{S_t} \frac{1 + \ln\left(S_t/K\right)}{K^2/2} P_t(\tau; K) \, dK. \tag{10}$$

where S_t is the price of underlying stock, and $C_t(\tau; K)$ and $P_t(\tau; K)$ are call and put prices with maturity τ and strike K, respectively. The risk-neutral expectation of the quadratic payoff is then

$$\mathbb{E}_{t}^{\mathbb{Q}}\left[r_{t,t+\tau}^{2}\right] = e^{r_{f}\tau}V_{t}\left(\tau\right),\tag{11}$$

where r_f is the continuously compounded interest rate.

We compute $V_t(\tau)$ for each firm on each day and by each days-to-maturity. In theory, computing $V_t(\tau)$ requires a continuum of strike prices, while in practice we only observe a discrete and finite set of them. Following Jiang and Tian (2005) and others, we discretize the integrals in equation (10) by setting up a total of 1001 grid points in the moneyness (K/S_t) range from 1/3 to 3. First, we use cubic splines to interpolate the implied volatility inside the available moneyness range. Second, we extrapolate the implied volatility using the boundary values to fill the rest of the grid points. Third, we calculate option prices from these 1001 implied volatilities using the formula of Black and Scholes (1973).⁷ Next, we compute $V_t(\tau)$ if there are four or more OTM option implied volatilities (e.g. Conrad, Dittmar, and Ghysels 2013 and others). Lastly, to obtain V_t (30) for a firm on a given day, we interpolate and extrapolate $V_t(\tau)$ with different τ . This process yields a daily time series of the risk-neutral expected quadratic payoff for each eligible firm in the sample.

Note that the price of the quadratic payoff $V_t(\tau)$ in equation (10) is the sum of a portfolio of

⁷We apply these steps to the estimation of the market and individual risk-neutral expected quadratic payoffs. Although the market options are European, the individual equity options are American. Therefore, directly using the mid-quotes of individual options is inappropriate because the early exercise premium may confound our results. To avoid this issue, we use the implied volatilities provided by OptionMetrics. These implied volatilities are computed using a proprietary algorithm based on the Cox, Ross, and Rubinstein (1979) model, which takes the early exercise premium into account.

OTM call options and a portfolio of OTM put options:

$$V_t(\tau) = V_t^g(\tau) + V_t^l(\tau), \qquad (12)$$

where:

$$V_t^l(\tau) = \int_0^{S_t} \frac{1 + \ln(S_t/K)}{K^2/2} P_t(\tau; K) \, dK \quad \text{and} \quad V_t^g(\tau) = \int_{S_t}^\infty \frac{1 - \ln(K/S_t)}{K^2/2} C_t(\tau; K) \, dK. \tag{13}$$

In Subsection A.1 of the Internet Appendix, we analytically prove that $V_t^g(\tau)$ is the price of the quadratic gain, and $V_t^l(\tau)$ is the price of the quadratic loss. Held, Kapraun, Omachel, and Thimme (2018) also provide proof to support this loss and gain decomposition. Hence, the risk-neutral expectation of quadratic loss and gain are:

$$\mathbb{E}_{t}^{\mathbb{Q}}\left[l_{t,t+\tau}^{2}\right] = e^{r_{f}\tau}V_{t}^{l}\left(\tau\right) \quad \text{and} \quad \mathbb{E}_{t}^{\mathbb{Q}}\left[g_{t,t+\tau}^{2}\right] = e^{r_{f}\tau}V_{t}^{g}\left(\tau\right). \tag{14}$$

While the risk-neutral expected quadratic payoff can be estimated from available option data following Bakshi, Kapadia, and Madan (2003), estimating the risk-neutral expected variance is empirically infeasible in a similar model-free fashion. As shown in equation (7), the risk-neutral expected realized variance is the sum of the risk-neutral expectations of squared high-frequency returns. To estimate these expectations, one needs observable options with high-frequency maturity δ or variance strikes in a variance swap market. However, high-frequency (daily or 5-min) maturing options are not traded and liquid variance swap markets only exist for a minority of large stocks and indices. By using QRP instead of VRP, we also alleviate the severe empirical limitations in computing risk-neutral expected realized variance, thus we can accommodate for a large crosssectional study with companies of various size.

3.2 Estimating the Physical Conditional Expected Quadratic Payoff

We use a regression model to estimate the expectations of squared monthly returns and the loss and gain components. We assume that, conditional on time-t information, monthly log returns $r_{t,t+1}$ follow a normal distribution with time-varying mean $\mu_t = \mathbb{E}_t [r_{t,t+1}]$ and time-varying variance $\sigma_t^2 = \mathbb{E}_t \left[RV_{t,t+1} \right]$. These expectations are therefore

$$\mathbb{E}_t \left[r_{t,t+1}^2 \right] = \mu_t^2 + \sigma_t^2 \quad \text{and} \quad \begin{cases} \mathbb{E}_t \left[l_{t,t+1}^2 \right] = \left(\mu_t^2 + \sigma_t^2 \right) \Phi \left(-\frac{\mu_t}{\sigma_t} \right) - \mu_t \sigma_t \phi \left(\frac{\mu_t}{\sigma_t} \right) \\ \mathbb{E}_t \left[g_{t,t+1}^2 \right] = \left(\mu_t^2 + \sigma_t^2 \right) \Phi \left(\frac{\mu_t}{\sigma_t} \right) + \mu_t \sigma_t \phi \left(\frac{\mu_t}{\sigma_t} \right), \end{cases}$$
(15)

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the standard normal density and cumulative distribution functions, respectively. An estimate of $\mu_t (= Z_t^{\top} \beta_{\mu})$ is the fitted value from a linear regression of monthly returns onto predictors Z_t , while an estimate of σ_t^2 is the fitted value from a linear regression of monthly total realized variances onto the same predictors Z_t . More specifically, $\mathbb{E}_t [RV_{t,t+1}] = \mathbb{E}_t [RV_{t,t+1}^g] + \mathbb{E}_t [RV_{t,t+1}^l]$, $\mathbb{E}_t [RV_{t,t+1}^g] = Z_t^{\top} \beta_{\sigma}^g$ and $\mathbb{E}_t [RV_{t,t+1}^l] = Z_t^{\top} \beta_{\sigma}^l$.

Predictors Z_t include the constant, and the loss (bad) and gain (good) realized variances of the past month (t-1 to t), the past five months (t-5 to t), and the past twenty-four months (t-24 to t). Our model is a variant of the HAR-RV model of Corsi (2009). While the original HAR-RV model is used to forecast daily realized variance, our variant model targets the monthly realized variance. In our forecasting regression, the loss and gain components of the realized variance are separate regressors to account for their asymmetric effects in return forecasting (e.g., Feunou, Jahan-Parvar, and Tédongap 2013; Bekaert, Engstrom, and Ermolov 2015; Patton and Sheppard 2015) and in volatility forecasting (e.g., Patton and Sheppard 2015). Prior studies provide strong evidence that decomposing the realized variance into its loss and gain components significantly improves the explanatory power of various HAR-RV models.

4 Data and Descriptive Statistics

4.1 Data

Our sample runs from January 1996 to December 2015. Data on individual stock and S&P 500 returns are from the Center for Research in Security Prices (CRSP). We keep two more years of returns (January 1994-December 1995) to compute the physical expectations of realized variance for the start of the sample. Following the literature on cross-section studies, we keep only common stocks listed on the NYSE, AMEX, and NASDAQ, which are firms that have CRSP share codes of

⁸Estimates of μ_t and σ_t^2 are consistent and unbiased quasi-maximum likelihood estimators. Diagnostic tests show that we can't reject the conditional normality assumption at the 5% significance level for the large majority of stocks.

10 and 11 and CRSP exchange code of 1, 2 or 3. In order to control for firm-level characteristics, we collect data on market capitalization (price times outstanding shares) and book values from CRSP and Compustat, respectively. For each firm, its size is computed as the log of market capitalization, and the firm's book-to-market ratio is its book value divided by its market capitalization.⁹ To gauge the performance of a stock in the past year, we compute the prior 12-month returns as the individual stock's cumulative excess returns from month t - 13 to t - 2 to avoid spurious effects. To control for market factors, we collect the data on the market excess returns (market returns in excess of the one-month T-bill rate), the size, value, and momentum factors from Kenneth French's data library.¹⁰ We also obtain data on VIX from the Chicago Board Option Exhange (CBOE).

For the estimation of the risk-neutral quadratic payoff, we rely on stock options (individual firm-level and S&P 500) obtained from the IvyDB OptionMetrics database. We exclude options with missing or negative bid-ask spread, zero bid, or zero open interest (e.g, Carr and Wu 2009). Following Bakshi, Kapadia, and Madan (2003), we restrict the sample to out-of-the-money options. To ensure that our results are not driven by misleading prices, we follow Conrad, Dittmar, and Ghysels (2013) and exclude options that do not satisfy the usual option price bounds, missing implied volatility, or options with less than 7 days to maturity. For a firm on a given day and a given maturity, we record the risk-neutral expected quadratic payoff as missing if there are less than four OTM implied volatilities. For details on the estimation methodology, see section 3.1.

To merge the option data with the CRSP stock data, we follow the approach in Duarte, Lou, and Sadka (2006). The size of the cross-section is mostly determined by the number of firms with available and eligible stock option data. In January 1996, the cross-section contains 426 firms, while in December 2015, the size of the cross-section has grown significantly to 1,245 firms. The average size of the cross-section throughout our sample period is approximately 898.

4.2 Descriptive Statistics

Our sample covers a wide range of firm size. We report descriptive statistics for firm-level characteristics in Panel A of Table 1. Median values of the loss, gain, and net QRPs are positive on

⁹Consistent with the literature, we remove firms with negative book values. Since book value is only observed yearly, the daily variability of the book-to-market comes solely from the changes in the market capitalization. Thus we may have extremely large book-to-market for distressed firms if these firms' market capitalization significantly decrease within days. Therefore, we winsorize the book-to-market ratio at the 99% level.

¹⁰http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html.

average, equal to 32.57, 13.85, and 13.82 in monthly percentage-squared units, respectively. The median value of stock illiquidity (ILLIQ) has a mean of 4.8e-3 with positive skewness and kurtosis, which are comparable to values reported in Amihud (2002). The median value of firm risk-neutral skewness (SKEW) is on average -0.51, which is in the same range as values reported by Conrad, Dittmar, and Ghysels (2013). The median value of firm idiosyncratic volatility (IVOL) is 2.04% on average, which also compares well with the findings of Hou and Loh (2016).

In Panel A of Table 1, we also show the descriptive statistics of market-wide factors. These factors are control variables in subsequent cross-sectional analyses of the relation between expected stock returns and QRP. The market loss QRP is on average 15.65 while the market gain QRP is much smaller on average equal to 2.92, which leads to a positive average value of 12.73 for the market net QRP.¹¹ Furthermore, the market loss and gain QRP have distinct dynamics. For instance, the market loss QRP exhibits twice the volatility of market gain QRP (14.32 verse 7.11); the kurtosis of market loss QRP is half the kurtosis of gain QRP (6.22 verse 12.58); and the market loss QRP is more persistent with a first-order autocorrelation coefficient of 0.79 compared to the gain QRP's much lower autocorrelation of 0.49. The market risk-neutral skewness is negative on average with a value of -1.96, consistent with the values reported in previous studies; for example, -1.26 in Bakshi, Kapadia, and Madan (2003).

Panel B of Table 1 shows the time series averages of the cross-sectional correlations between firm-level variables. Since the net QRP is the difference between the loss and gain QRP, as expected, the net QRP is positively correlated with the loss QRP and negatively correlated with the gain QRP in the cross-section, with correlation values of 0.87 and -0.61, respectively. The loss QRP and the gain QRP have a cross-sectional correlation of -0.20. Interestingly, the QRP measures show little cross-sectional correlations with other firm characteristics such as the stock illiquidity, risk-neutral skewness, idiosyncratic volatility, etc. The absolute correlation values do not exceed 0.15 in general. This suggests that we can rule out potential multicollinearity issues that may affect statistical inference in subsequent empirical tests; for example, in cross-sectional regressions of excess returns on quadratic risk premium and other firm characteristics.

¹¹For comparison, the mean of market total VRP as reported by Bollerslev, Tauchen, and Zhou (2009) is 18.30. The relatively smaller average value of gain QRP also suggests that the average investor is relatively indifferent about fluctuations in market gain uncertainty, although she does care about fluctuations in market loss uncertainty. In a sense, these statistics also corroborate the findings of Feunou, Jahan-Parvar, and Okou (2018), who shows that the market bad VRP is the most important component of the market total VRP.

In Panel A of Figure 2, we plot the market loss QRP on the right y-axis and the month-bymonth cross-sectional median values of firm-level loss QRP on the left y-axis. All variables are reported in monthly percentage-squared units. In all months, both the market loss QRP and the median firm loss QRP are nonnegative. They both peak during large market downturns. In particular, the market loss QRP peaked at 61.80 in September 1998 during the long-term capital management (LTCM) crisis, 61.11 in September 2002 toward the end of the dot-com bubble, and 80.02 in January 2009 during the recent financial crisis. Similarly, in Panel B, we plot the market gain QRP, together with the month-by-month median values of firm-level gain QRP. In roughly two thirds of our sample period, the market gain QRP is positive. On the other hand, the median firm gain QRP is above zero in almost all periods of our sample with few exceptions. Among these crisis periods, the market gain QRP peaked at 14.52 in September 1998 during the LTCM crisis, and 47.74 in October 2008 during the recent financial crisis.

5 Results

We now provide an empirical assessment of the cross-sectional relationships between the reward for investing in stocks (measured by their expected excess returns), and the stock's downside and upside risks (measured by their loss QRP and gain QRP, respectively). We assess these relationships through portfolio sorts and cross-sectional regressions. Since the loss and gain QRP have little cross-sectional correlation as shown in Panel B of Table 1, we start by studying univariate sorted portfolios based on these QRPs. Next, we pair up each of our QRPs with each of the control variables investigated in the literature in bivariate portfolio sorts. These two-dimensional sorts are useful to examine QRPs' additional cross-sectional predicting power beyond existing variables. Finally, we run firm-level cross-sectional regressions to jointly estimate the prices of risks associated with the loss and gain QRP, when controlling altogether for multiple cross-sectional effects.

5.1 Single Sorting

We first analyze univariate portfolio sorts involving our estimates of firm-level QRPs. More specifically, at the end of each month, we sort firms into quintiles based on their corresponding monthly average values for the specific characteristic, such as the loss, gain or net QRPs. Quintile 1 thus contains the firms with QRP values in the bottom 20% while quintile 5 contains firms with QRP values in the top 20%. Then, for each quintile we use end-of-month market capitalizations to form a value-weighted portfolio and measure its excess returns over the next month.¹² For each quintile, we report the cross-sectional average value of a specific characteristic (the loss, gain or net QRPs), as well as the portfolio average monthly excess returns and alphas, where alphas are computed relative to the five-factor model of Fama and French (2015).

Panel A of Table 2 shows that there is a wide range of loss QRP values among our quintile sorted portfolios based on the loss QRP. The time-series average of loss QRP are -145.96 and 231.63 for the lowest and highest quintiles, respectively. Similarly, Panel B of Table 2 shows that the gain QRP values, among our gain QRP sorted portfolios, also cover a wide range from a low of -59.68 to a high of 163.99 on average. Take the two lowest quintiles for example, as discussed in Subsection 2.2, the lowest quintiles consists of firms which are either associated with weak downside risk (measured by its loss QRP) or immense upside potential (measured by its gain QRP) in bad times, in contrast to firms in the highest quintiles, respectively.

Turning to the cross-sectional pricing effect of the QRP components (the loss, gain or net QRPs), we present the portfolio average monthly excess returns and alphas in Table 2. In Panel A of Table 2, when firms are sorted based on their loss QRPs, the average excess returns and alphas are monotonically increasing from the lowest quintile to the highest quintile. The average monthly excess returns of the lowest quintile is -0.98% which is significantly lower than the average value 2.10% for the highest quintile portfolio, resulting a high-minus-low difference of 3.08% per month on average. Beyond that, the risk-adjusted performance measured by portfolios' alpha confirms that on average the highest quintile portfolio is better remunerated than the lowest quintile portfolio. The high-minus-low portfolio has a alpha of 2.79% per month with a *t*-statistic equal to 6.82 which is significant at the 99% confidence level. As discussed in Subsection 2.2, investors are risk-averse and prefer firms with lower loss QRP because these firms' downside risk tend to disappear in bad times. Therefore investors are happy to face less or no insurance costs and they are willing to pay more for such assets, thus accepting a lower premium to invest in them. In contrast, firms with higher loss QRP are often disliked by investors since these firms' downside risk tend to be severe

¹²Measuring post-ranking excess returns in portfolio sorts avoids spurious effects (e.g., Fama and French 1993; Ang, Hodrick, Xing, and Zhang 2006; Chang, Christoffersen, and Jacobs 2013).

in bad times. As a result, investors incur more insurance costs and they are willing to pay less for such assets, thus requiring a larger premium.

In Panel B of Table 2, when sorting with respect to gain QRP, we also find that average excess returns are monotonically increasing from the lowest quintile to the highest quintile portfolio. The average monthly excess returns of the lowest quintile is -0.97% which is significantly lower than the average value 1.95% for the highest quintile portfolio, resulting a high-minus-low difference of 2.95% per month on average. Beyond that, the risk-adjusted performance measured by portfolios' alpha again confirms that on average the top quintile portfolio performs better than the bottom quintile portfolio does. The high-minus-low portfolio has a alpha of 2.78% per month with a *t*-statistic equal to 7.94 which is significant at the 99% confidence level.

Following our discussion in Subsection 2.2, investors are potential-seeking and prefer firms with lower gain QRP since these firms' upside potential tend to be strong in bad times. Therefore, investors require less or no protections and they are then willing to pay more for such assets, thus accepting a lower premium. To the contrary, investors dislike firms with higher gain QRP since these firms' upside potential shrink in bad times. This leads to a larger required compensation for such assets thus a higher premium.

Panel C of Table 2 shows results when firms are sorted on their net QRPs — the difference between loss and gain QRPs. The high-minus-low average excess returns and alphas are much smaller compared to sorting on the loss or gain QRP, and not statistically significant at conventional levels. This suggest that, although the premium on loss and gain uncertainty is highly relevant for the cross-section of expected stock returns, the premium for the net effect is not. Therefore, it is crucial to decompose the total uncertainty of a stock into its loss and gain components.

To summarize, the loss QRP and the gain QRP generate monotonic patterns in the average returns of sorted portfolios with statistically significant differences between the highest and the lowest quintiles. Sorting firms on their loss QRPs leads to a somewhat larger heterogeneity in performance than sorting firms on their gain QRPs. On the other hand, the net QRP does not generate monotonic trends in returns or alphas, and we find no evidence that it is priced in the cross-section of expected stock returns.¹³ These results suggest that the loss QRP and the gain

¹³The results also hold for value-weighted tercile and decile portfolios, as well as for equally-weighted portfolios. These untabulated results are available upon request.

QRP contain different information contents, and it is crucial to consider these two QRP components separately for cross-section of expected stock returns.

5.2 Double Sorting

We now examine whether variations in QRP components (the loss and gain QRPs) are subsumed by various cross-sectional effects discussed in the extant literature. Following Fama and French (1992), we first sort firms into five groups based on a key variable (systematic risk or characteristic) representing a specific cross-sectional effect documented in the literature. Next, within each group, we further sort firms into quintile portfolios based on each QRP component. If the information content of the QRP component had no additional value for investors, then average excess returns on quintile portfolios from the second sorts based on the QRP component would not generate a significant high-minus-low difference. For the second sorts, we report the average difference of the high-minus-low ("5-1") excess returns, together with the corresponding t-statistic. Here the highest quintile "5" contains firms with the highest QRP component and the lowest quintile "1" contains firms with the lowest QRP component.

5.2.1 Controlling for Systematic Risk Measures

We control for systematic risk measures that are motivated by leading asset pricing models and financial theories. Since our loss and gain QRPs have asymmetric effects on cross-section of expected stock returns, we start by considering downside risk measures which are also motivated by this asymmetric treatment.¹⁴ Farago and Tédongap (2018) prove that in an intertemporal equilibrium asset pricing model featuring generalized disappointment aversion (GDA) and changing macroeconomic uncertainty, besides the market return and market volatility, three downside risk factors are also priced: a downstate factor, a market downside factor, and a volatility downside factor.¹⁵ These five GDA factors depend on two variables: the log market return and changes in

¹⁴The asymmetric treatment of loss and gain has a long standing in the academic literature (see for example Roy, 1952 and Markowitz, 1959) and has motivated the development of theories of rational behavior under uncertainty that imply priced downside risk in capital markets (see for example Bawa and Lindenberg, 1977, Kahneman and Tversky, 1979, Quiggin, 1982, Gul, 1991, and Routledge and Zin, 2010).

¹⁵Empirical studies by Ang, Chen, and Xing (2006) and Lettau, Maggiori, and Weber (2014) examine the pricing of market downside risk as motivated by the disappointment aversion theory of Gul (1991), for several asset classes. More recently, Farago and Tédongap (2018) show that in the presence of fluctuating macroeconomic uncertainty, volatility downside risk is priced in addition to market downside risk, and their findings give strong support to the

market conditional variance. To measure the unobservable market conditional variance, we use σ_t^2 estimated in Section 3.2. Following Farago and Tédongap (2018, see their Online Appendix), we use short-window regressions to estimate the stocks' exposures to the GDA factors. Details are provided in Subsection A.2 in the Internet Appendix.

Table 3 shows the results of the double sorts when we control for exposures to these five GDA factors in the first five panels. Note that these five exposures are obtained from the same regression all together, while double sorts pair up each of these exposures with a QRP component one at a time. In Panel A, firms with high loss QRP outperform those with low loss QRP within all five groups of each of the GDA factor exposures. For instance, when we control for exposures to the three downside risk factors, the sizeable high-minus-low spreads range between 2.14% and 4.08% per month. Likewise in Panel B, firms with high gain QRP outperform those with low gain QRP within all five groups of each of the GDA factor exposures, with sizeable spreads ranging between 1.96% and 4.54% per month when controlling for exposures to the three downside risk factors of the GDA model. All reported spreads are statistically significant at the 95% or higher confidence level. This suggests that the cross-sectional variation in average excess returns reflects the heterogeneity in firm QRP components that is unrelated to heterogeneous exposures to leading downside risk measures across stocks.¹⁶ We finally observe from Table 3 that patterns of the alphas are very similar to patterns of the expected excess returns across the different quintile portfolios.

We consider three other systematic risk factors for which variations are likely correlated with firm-level QRP components, namely the market loss and gain QRPs (see Figure 2), and the market risk-neutral skewness. The choice of market QRP components is motivated from the consumptionbased general equilibrium asset pricing model proposed by Bollerslev, Tauchen, and Zhou (2009) featuring time-varying risk in the stochastic volatility. Their model suggests three cross-sectional pricing factors: market excess returns, innovations in market conditional variance, and innovations in market variance of variance. We substitute the variance of variance factor with the market loss and gain QRPs and measure firm exposures to these two market QRP components from the resulting four-factor model.¹⁷ Lastly, firm exposures to the market risk-neutral skewness is

generalized version of the disappointment aversion theory as developed by Routledge and Zin (2010).

¹⁶We focus on the work of Farago and Tédongap (2018) when controlling for existing downside risk measures, as the authors prove theoretically that the downside risk measures in Ang, Chen, and Xing (2006) and Lettau, Maggiori, and Weber (2014) are particular linear combinations of the multivariate GDA factor exposures.

¹⁷Since the model in Bollerslev, Tauchen, and Zhou (2009) also implies that the market VRP is solely determined

calculated following Chang, Christoffersen, and Jacobs (2013). Details are provided in Subsection A.2 in the Internet Appendix.

Table 3 also displays double-sorting results on firm QRP components when we control for exposures to market QRP components and the market risk-neutral skewness. As shown in both panels, controlling for exposures to either market QRP components or the market risk-neutral skewness does not hinder the ability of firm QRP components to explain cross-sectional differences in average excess returns. Firms with high loss QRP outperform those with low loss QRP within all five clusters of each of the exposures to the market loss QRP and the market risk-neutral skewness, with sizeable spreads ranging between 2.12% and 4.60% per month. Likewise, firms with high gain QRP outperform those with low gain QRP within all five clusters of exposures to the market gain QRP and the market risk-neutral skewness. These spreads range between 1.96% and 4.54% per month. All reported spreads are statistically significant at the 95% or higher confidence level.

Altogether, these results suggest that the cross-sectional variation in average excess returns reflects heterogeneity in firm QRP components that is unrelated to the heterogeneous exposures to various systematic risk across stocks. The systematic risk factors considered here includes the five GDA factors, the market loss and gain QRPs and the market risk-neutral skewness.

5.2.2 Controlling for Other Firm Characteristics

We again use double-sorting methodology to examine whether the asset pricing information in some major firm characteristics already account for the pricing information embedded in firm QRP components.¹⁸ If firm QRP components were priced simply because they capture the information content of other firm characteristics, then controlling for these other firm characteristics would yield a weak or insignificant cross-sectional variation in average returns across stocks sorted on firm QRP components. In Subsection A.2 in the Internet Appendix, we provide details about the source and construction method for the time series of the firm-level characteristics we control for.

First, we control for the implied volatility smirk proposed by Xing, Zhang, and Zhao (2010) and Yan (2011). The authors define the implied volatility smirk as the difference between the implied

by the variance of variance, and given the bias in measuring VRP and its components in the literature (see the discussion in 2.3), we choose to use our loss and gain QRPs instead.

¹⁸We treat QRP components (the loss, gain and net QRPs) as firm characteristic because there are no observable market-wide factors such that QRP components measure the associated systematic risk exposures (or factor loadings).

volatility of out-of-the-money (OTM) puts and at-the-money (ATM) calls. They show that the implied volatility smirk is a strong predictor of expected returns in the cross-section because it captures a stock's tail risk. We compute it for all the firms in our sample. Although both implied volatility smirk and loss QRP are measuring downside risk, we find that the average cross-sectional correlation between these two measures is 0.12. This suggests that the implied volatility smirk and the loss QRP are capturing different information about the downside risk of a stock.

Table 4 presents results when we sort stocks by their QRP components after controlling for the implied volatility smirk (SKEW thereafter). Both panels show that firms with high loss (gain) QRP outperform those with low loss (gain) QRP within all five groups of SKEW, with sizeable spreads ranging between 2.45% (2.40%) and 4.02% (4.27%) per month. All reported "5-1" spreads are statistically significant at the 95% or higher confidence level. We obtain similar findings for other measures capturing firm-level downside risk such as the risk-neutral skewness (Rehman and Vilkov, 2012; Conrad, Dittmar, and Ghysels, 2013; Stilger, Kostakis, and Poon, 2016; Bali, Hu, and Murray, 2019) and the physical skewness as measured by the relative signed jump variation (Bollerslev, Li, and Zhao, forthcoming).¹⁹ These results show that the cross-sectional variation in average excess returns reflects heterogeneity in firm QRP components that is unrelated to heterogeneity in various firm-level downside risk stocks.

Beyond firm characteristics capturing the downside risk, other characteristics we control for in Table 4 include the idiosyncratic volatility (Ang, Hodrick, Xing, and Zhang, 2006), the stock illiquidity (Amihud, 2002), the analysts' coverage of the stock as proxied by the number of analysts (Hong, Lim, and Stein, 2000), and the demand for lottery as proxied by the maximum daily return during the previous month (Bali, Cakici, and Whitelaw, 2011). After controlling for these firm characteristics, there is still a positive and significant cross-sectional relation between QRP components and expected returns.²⁰ We find that the spreads range between 0.94% and 5.84% per month and they are all significant at the 95% or higher confidence level.²¹

¹⁹In untabulated results, we also control for the risk-neutral counterpart of the relative signed jump variation (Huang and Li, 2019), and find that our main results hold.

²⁰We also investigate if the volatility spread (Bali and Hovakimian, 2009; Cremers and Weinbaum, 2010) or option illiquidity (Goyenko, Ornthanalai, and Tang, 2015) subsume the predictability by QRP components. We report results for conditional double-sorts on the volatility spread or option illiquidity and QRP components in Table B1 of the Internet Appendix. We find that the QRP components are still strongly significant after controlling for either the volatility spread or option illiquidity.

²¹We note that our double-sort results do not imply that cross-sectional predictability by QRP components sub-

5.3 Fama-MacBeth Regressions

In this subsection, we follow the procedure introduced by Fama and MacBeth (1973) and run monthby-month cross-sectional regressions using individual firms. These cross-sectional regressions allow us to estimate the sensitivity of expected returns to stock QRP components—prices of downside and upside risks associated with loss and gain QRP, respectively. Through cross-sectional regressions, we can also control for various cross-sectional effects at once. While using portfolios as test assets in Fama-MacBeth regressions is fairly common, our choice of individual stocks follows Ang, Liu, and Schwarz (2010) and Gagliardini, Ossola, and Scaillet (2016), who highlight the advantage of the use of a large cross-section of individual stocks versus a few portfolios. They find that using portfolios destroys important and necessary information, which leads to much less efficient estimate of the cross-sectional risk prices. Other than the efficiency gain, using individual stocks as test assets will also yield more conservative estimates.

In Table 6, we report the time series average of the risk prices of QRP components, where we control for systematic risk in Fama-MacBeth regressions. There are seven different model specifications. In Model I, the net QRP is used to explain differences in the expected returns. The estimated average prices of the net QRP is 0.07 with *t*-statistic equal to 1.60, which is not statistically significant at conventional levels.²² In Model II, we use both the loss QRP and the gain QRP separating the downside risk from the upside risk. The price of the loss QRP (measuring the downside risk) is 0.616 with *t*-statistic equal to 8.90, and the price of the gain QRP (measuring the upside risk) is 1.348 with *t*-statistic equal 12.06.²³ Both effects are statistically significant at the 99% confidence level. These results show that decomposing the net QRP into two components (loss QRP and gain QRP) proves meaningful in the Fama-MacBeth regressions.²⁴

sumes the predictability by the other firm characteristics or factor exposure, which have been shown to have significant predictive power on the cross-section of expected excess returns across all stocks in CRSP for different sample periods and horizons. Our sample includes only optionable stocks, and covers a different sample period. For example, untabulated monthly univariate sorts based on firms' exposure to market risk-neutral skewness or firm's relative signed jump variation yield statistically insignificant spreads in our sample.

 $^{^{22}}$ Harvey, Liu, and Zhu (2016) show that any new factor needs to have a *t*-statistic greater than 3.0. While the net QRP is a firm characteristic and not a factor, we still believe the hurdle is relevant.

 $^{^{23}}$ These t-statistics may look high but they are in line with previous literature on other anomalies such as e.g. the asset growth anomaly (Cooper, Gulen, and Schill, 2008). Further, if we run cross-sectional regressions of month t + 3 or month t + 12 excess returns on month t loss and gain QRP, we find that both the t-statistics and coefficients decrease significantly. These results can be found in Tables B2 to B5 in the Internet Appendix.

²⁴This is similar to the findings of Campbell and Vuolteenaho (2004) and Bansal, Dittmar, and Lundblad (2005). Starting from the CAPM and the consumption-based CAPM, respectively, the authors decompose total asset risk into a cash flow component and a discount rate component. They find weak evidence that total asset risk is priced,

We design five other models (Models III to VII) to test the robustness of Model II. After controlling for the CAPM beta (Model III), firm exposure to market skewness (Model IV), firm exposures to market variance and market QRP (Model V), Carhart factor exposures (Model VI), and GDA factor exposures (Model VII). The statistical significance and economic magnitude of the prices of loss and gain QRPs remain unchanged. This suggests that cross-sectional predictability by QRP components is not subsumed by exposures to existing systematic risk factors.

We now turn to the Fama-Macbeth results in Table 7, where we control for other firm characteristics in Model VIII and IX. In Model VIII, we add the relative signed jump variation RSJ, while in Model IX we further include a considerably large panel of other firm characteristics including the idiosyncratic volatility (IVOL), size, book-to-market (B/M), illiquidity (ILLIQ), the risk-neutral skewness (FSKEW), realized semi-variances (RV^l and RV^g), short-term reversal (P01M) and momentum (P12M). Once again, accounting for these multiple cross-sectional effects does not erode the statistical significance or economic magnitudes of the prices of QRP components.

In summary, neither the systematic risk nor other firm characteristics appear to drive out either QRP component of the net QRP. The estimated prices of the loss (gain) QRP range from 0.609 (1.348) to 0.848 (1.677) in Tables 6 and 7. Since the time series average of the cross-sectional standard deviations of loss (gain) QRP is 270.32 (143.04), a one-standard-deviation increase in the loss (gain) QRP is associated with a 1.7%–2.3% (1.9%–2.4%) rise in monthly expected stock returns. These effects are highly economically significant. In contrast, since the average standard deviation of net QRP is 333.41, a one-standard-deviation increase in the net QRP is only associated with a 0.23% rise in monthly expected stock returns.

5.4 Robustness Checks

We perform a number of additional checks to verify the stability of our findings. All these results are in the Internet Appendix.

although have strong evidence for priced cash flow risk. Given these findings, Bansal, Dittmar, and Lundblad (2005) argue that, when multiple sources of risk are priced, solely using the combined exposure in cross-sectional regression can produce a "tilt," and the estimated price of risk can be insignificant. If, however, one extracts the different components of risk, then they should appropriately measure differences in risk premia attributable to the differences in the risk premia across assets, which the loss and gain QRP may explain.

Subsample Analysis We repeat the univariate sorts for two subsamples: one excludes the recent financial crisis (January 1996 - December 2006), and another excludes the IT-crisis (January 2003 - December 2015). We report the results in Table B6 in the Internet Appendix. These results confirm that the significant and positive cross-sectional relation between expected stock returns and the loss and gain QRP is not driven by the two crisis periods in recent years. We also perform the univariate sorts for the sample free from non-synchronicity of option and stock market (April 2008 - December 2015). We find the cross-sectional return predictability of both the loss and gain QRP remain strong.

Alternative Measures To gauge the robustness of our findings to alternative measures of QRP, we consider QRP components standardized either by the physical or risk-neutral expected quadratic payoff,²⁵ and the empirically feasible, yet potentially biased measure of the variance risk premium \widetilde{VRP} discussed in section 2.3. These results can be found in Tables B7 to B9 on the Internet Appendix. In general, all of our results hold using these three measures.

Dividend and Non-Dividend Paying Stocks We compute option prices assuming no dividend payments during the maturity period of an option. This is because dividends are hard to predict thus the large measurement errors in the predicted dividends may confound our results. However, due to the zero dividend assumption, firms which are expected to pay dividends have underpriced put option prices and overpriced call option prices leading to a downward bias in their loss and gain QRPs.²⁶ We follow Cao and Han (2013) and analyze univariate sorts based on the loss and gain QRP for non-dividend paying stocks and dividend paying stocks separately in Table B10 in the Internet Appendix. In both subsamples, the predictability of QRP components is positive and significant. This predictability is much stronger in the subsample of the non-dividend than the subsample of the dividend paying stocks.

In summary, our results confirm that the loss and gain QRPs are significant and robust risk measures in the cross-section. In particular, while the downside risk has been shown to be priced

²⁵There is a large heterogeneity of QRP levels across stocks. For our cross-sectional empirical analysis, we observe stocks that have a relatively high or low QRP because their overall level of the expected quadratic payoff (risk-neutral or physical) is high or low. To address this issue, we follow Bollerslev, Li, and Zhao (forthcoming) and standardize QRP by the risk-neutral or physical expected quadratic payoff, respectively.

²⁶This potential bias is not well known in the literature, but is briefly discussed in Cao and Han (2013) and more recently in Branger, Hülsbusch, and Middelhoff (2018).

in previous literature, there is little evidence about the pricing of the upside risk. In this respect, our findings regarding the gain QRP complement the existing literature.

6 Discussion

The previous sections provide extensive and robust evidence that QRP components are strong and economically significant predictors of expected stock returns in the cross-section. We also find that "5-1" spreads on QRP components in double-sort results of Section 5.2 show significant discrepancy, sometimes a strong monotonic pattern, across the different quintiles of some the controlled firm characteristics. The larger the "5-1" spreads the stronger the cross-sectional predictability. Motivated by these monotonic patterns in spreads, we investigate possible explanations for the return predictability of QRP components. Because the predictability of QRP components is strongest among certain types of stocks relative to other categories, we argue that the underlying particular firm characteristic of these stocks might then be explaining their cross-sectional predictability. This section ends by discussing how QRP components can enhance our understanding of existing cross-sectional findings regarding the implied volatility smirk and idiosyncratic volatility.

Limits to Arbitrage In general, highly illiquid stocks are more costly (require higher capital) to arbitrage, thus they carry a higher risk. This would limit rational arbitrageurs in exploiting any arbitrage opportunity among these stocks. If this type of limit to arbitrage (e.g., Shleifer and Vishny 1997) is driving the predictability of the loss or gain QRP, we expect to find stronger predictability in the most illiquid stocks. Controlling for the stock illiquidity in Table 4, we find that both the loss and gain QRPs have the highest predictability among the most illiquid stocks, and this predictability decreases monotonically as the liquidity increases. Firms with high loss (gain) QRP significantly outperform those with low loss (gain) QRP within all quintiles. Most notably, among highly illiquid firms, the "5-1" spreads are almost three times as large as that for the most liquid firms on average. These results suggest that limits to arbitrage are in part driving the predictability of the loss and gain QRPs.²⁷

²⁷The idiosyncratic volatility also empirically characterizes the arbitrage risk (e.g., Ali, Hwang, and Trombley 2003; Cao and Han 2016). Similarly, we find that the predictability by the loss and gain QRPs also increases monotonically as the idiosyncratic volatility increases in Table 4.

Information Asymmetry Difficulties in interpreting downside and upside risk signals from the loss and gain QRPs may lead to potential asymmetric information among firms. Hong, Lim, and Stein (2000) use larger analyst coverage as an indicator of less information asymmetry, as higher analyst coverage means more diffusion of firm-specific information. We ask whether the strong return predictability by the loss and gain QRPs is in part reflecting the degree of asymmetric information among firms. If this is the case, we would expect to find the strongest predictability among firms with the highest degree of information asymmetry (lowest analyst coverage). Controlling for the average number of analysts covering the stock in Table 4, we find a monotonic pattern in "5-1" spreads. As analysts' coverage increases, the predictability by the loss and gain QRPs decrease and the predictability is the strongest among stocks with the highest information asymmetry (lowest analyst coverage). These results provide evidence that the information asymmetry partly drives the predictability by the loss and gain QRPs.²⁸

Demand for Lottery Kumar (2009), Bali, Cakici, and Whitelaw (2011), and Han and Kumar (2013) document that investors have a preference for lottery-like assets. Bali, Cakici, and Whitelaw (2011) show that a proxy for lottery demand (MAX) defined as the average of the five highest daily returns in a given month is negatively related to expected stock returns in the cross-section. If the predictability by the QRP components is partly driven by the investor demand for lottery-like features, this predictability should be the strongest (weakest) among stocks with high (low) MAX. Controlling for the stock MAX in Table 4, we find that firms with high QRP components outperform those with low QRP components within all quintiles of MAX. Notably, we find a monotonically increasing pattern in the "5-1" spreads. As the demand for lottery-like features increases, the predictability by the loss and gain QRPs increases significantly. The "5-1" spreads are more than three times higher among firms in the highest MAX quintile compared to those in the lowest MAX quintile. These results show that investors' demand for lottery-like features in part driving the strong predictability by the QRP components.²⁹

²⁸In Table B11 and B12 of the Internet Appendix, we find that the predictability by the loss and gain QRPs is highest among small firms, and it decreases as the firm size increases. To the extent that larger firms have less information asymmetry than smaller firms (e.g., Hong, Lim, and Stein, 2000; Bollerslev, Li, and Zhao, forthcoming), these results confirm that the information asymmetry may partly explain the predictability of QRP components.

²⁹In Table B13 of the Internet Appendix, we find that the predictability by the gain QRP is highest among growth firms, and decreases as the book-to-market ratio increases. To the extent that growth firms are mostly attractive for their upside potentials or their lottery-like feature relative to value firms, these results further confirm that the

Decrypting Implied Volatility Smirk versus Loss QRP In Panel A of Table 5, we further investigate whether the SKEW and the loss QRP measure different aspects of a stock's downside risk. We use a triple-sorting strategy to investigate the effect of SKEW within different levels of QRP components. Notably, we find some evidence that the cross-sectional predictability by SKEW is significant only among stocks with high loss QRP, and within this group, it is the strongest among stocks with high gain QRP. Firms with high loss QRP are the ones with high downside risk, while firms with high gain QRP are the ones with low upside potential. These firms are more likely to have expensive OTM put options and cheap ATM call options, and, thus, have the steepest implied volatility smirk. These findings otherwise confirm, yet complement the results of Xing, Zhang, and Zhao (2010) and Yan (2011).

Do QRP Components Rationalize the Idiosyncratic Volatility Puzzle? In Panel B of Table 5, we examine whether the QRP components may enhance our understanding of the idiosyncratic volatility (IVOL thereafter) puzzle. The IVOL puzzle was first documented by Ang, Hodrick, Xing, and Zhang (2006) and has become a popular asset pricing anomaly in the literature. Stambaugh, Yu, and Yuan (2015) find that IVOL is negatively priced among overpriced stocks, and has the highest predictability among overpriced stocks that are also difficult to short. We use a triple-sorting strategy to investigate the effect of IVOL within different levels of QRP components. Our findings suggest that the cross-sectional return predictability of IVOL is significant only among stocks with low loss QRP, and within this group, it is the strongest among stocks with low gain QRP. Stocks with low loss QRP are desirable to investors because their downside risk is low during bad times. Such stocks are in high demand and are potentially overpriced. Among them, stocks with low gain QRP are even more preferred by investors and shorting them is very risky and costly because their upside potentials tend to be strong in bad times. Thus stocks with both low loss QRP and low gain QRP are relatively expensive and are likely associated with difficulty to short. Our results directly relate to the findings reported in Stambaugh, Yu, and Yuan (2015), and extends their results using our measures of downside risk (loss QRP) and upside risk (gain QRP) to a large sample of optionable stocks.³⁰

lottery demand may partly explain the predictability of the QRP components.

³⁰Reading between the lines, the connection of our findings to the Stambaugh, Yu, and Yuan's results points to a link between QRP components and stock overpricing/underpricing which in practice can be appreciated through

Trading costs The average excess returns and alphas from the value-weighted single sorts based on either loss or gain QRP may seem relatively high.³¹ One might wonder whether these high alphas can be achieved among firms that are easier to trade (liquid firms with low limits to arbitrage) and tentatively have lower trading costs. In Table 4 we see that among the most liquid stocks, the alpha decreases by 32.2% to 1.89% per month (*t*-statistic of 3.85). Next, we see that among stocks where there are low limits to arbitrage (i.e., low idiosyncratic volatility), we similarly find that the alpha decreases by 65.2% to 0.97% per month (*t*-statistic of 3.35). Therefore, indirectly accounting for trading costs decreases the profitability of the trading strategy based on loss and gain QRP to levels comparable to other well-studied anomalies (e.g. Ang, Hodrick, Xing, and Zhang, 2006 find that sorting stocks by idiosyncratic volatility yields a spread of -1.38%). Further, the average turnover rate of the loss (gain) QRP quintile portfolios is 51.5% (52.0%). Following Jegadeesh and Titman (1993), we account for trading costs by considering a 0.5% one-way transaction cost. This means that the long-short portfolio alphas based on loss (gain) QRP decrease to 2.28% (2.26%) after accounting for transaction costs.

Microstructure effects Another possible explanation for the high spreads in our single-sorts is that loss and gain QRP are in part related to microstructure effects that are not truly tradable. Thus, once controlling for these effects, the spreads will shrink. To examine this possibility, we follow Ang, Hodrick, Xing, and Zhang (2006) and look at alternative trading strategies based on loss or gain QRP. The portfolio formation strategies follow Jegadeesh and Titman (1993) and are based on an estimation period of L months, a waiting period of M months, and a holding period of N months, together forming the L/M/N strategy. The main results in our paper are based on the 1/0/1 strategy, in which we sort stocks into quintile portfolios based on their average level of loss or gain QRP in month t, and then, for each quintile we use end-of-month market capitalizations to

valuation ratios such as book-to-market. At times, growth stocks may be seen as expensive and overvalued, as they are generally perceived by investors as stocks with large upside potential. Consistent with that view, we find that gain QRP has the highest predictability among firms with low book-to-market ratio. To the contrary, some investors may prefer value stocks which are generally perceived as undervalued by the market. These investors are however aware of the large downside risk of value firms, induced by operating leverage (see for example Garía-Feijóo and Jorgensen, 2010 and Hsieh and Lee, 2010). Consistent with that view, we find that loss QRP has the highest predictability among firms with high book-to-market ratio. These results are available in Table B13 in the Internet Appendix.

 $^{^{31}}$ Previous literature on other anomalies also finds high average excess returns, e.g. Cooper, Gulen, and Schill (2008) find that sorting firms by their asset growth yield monthly excess returns on the long-short portfolio of -1.73% (*t*-statistic of -8.45). Yan (2011) sorts stocks based on their implied volatility slope and finds risk-adjusted returns for a long-short portfolio of 1.9% per month (*t*-statistic of 9.64).

form a value-weighted portfolio and measure its excess returns over month t+1. In Table B14 of the Internet Appendix, we report results for two different waiting periods, in which we form portfolios based on their average loss or gain QRP in month t - 1 or month t - 3, respectively. We find that if we increase the waiting period to 1 (3) months, the spread for loss QRP decreases by 31.5% (34.8%) to 1.91% (1.82%) with a t-statistic of 5.19 (5.58). Similarly, for gain QRP we find that the spread decreases by 18.7% (45.3%) to 2.26% (1.52%) with a t-statistic of 6.30 (4.16). These results show that the predictability of loss and gain QRP is in part driven by microstructure effects that are not truly tradable. Once we account for these microstructure effects, we find spreads that are comparable to other well-known anomalies.³²

To summarize, the cross-sectional predictability by the loss and gain QRP is strong and reinforced among certain categories of stocks. It should however be noticed that, while at the first glance the spreads on the loss and gain QRP sorting strategies may seem high, once we indirectly account for various market frictions, we find that these spreads are more in line with previous literature on well-known anomalies.

7 Conclusion

We decompose the quadratic payoff on a stock into its loss and gain components and measure the premia associated with their fluctuations using stock and option data from a large crosssection of firms. The quadratic risk premium (QRP), defined as the difference between the riskneutral and physical expectations of quadratic payoff, represents the premium paid to insure against fluctuating loss uncertainty (loss QRP), net of the premium received to compensate for fluctuating gain uncertainty (gain QRP). Thus, the loss QRP measures the downside risk while the gain QRP measures the upside risk of an individual firm.

We show that the heterogeneity in the loss and gain QRPs across stocks is associated with differences in expected returns in the cross-section. Our findings suggest that expected stock returns in the cross-section are positively related to the loss and gain QRPs. On the other hand, we find no evidence of a cross-sectional relation between the net QRP and expected returns. Sorting stocks into portfolios based on their individual loss (gain) QRP results in an economically large

 $^{^{32}}$ In Table B15 of the Internet Appendix, we present single-sort results after removing microcaps from our sample (stocks with beginning of month price less than 5 USD). The spreads are virtually unchanged from Table 2.

monthly expected return spread between the stocks in the highest and lowest quintiles of 3.08% (2.95%). The return spreads remain highly statistically significant and economically important in double-sorting strategies and in Fama and MacBeth (1973) regressions controlling for exposures to various systematic risk factors and other firm characteristics.

In particular our result regarding the gain QRP shows that the upside risk is significantly and robustly priced even after the downside risk has already been accounted for. Since there is little evidence in the literature about the pricing of the upside risk, this result on gain QRP constitutes an important contribution.

Crucially, our results suggest that when analyzing the relation between expected stock returns and individual firm QRP, it is imperative to decompose the QRP into its loss and gain components. An interesting extension of our empirical analysis would be to expand the cross-section to include international firms. Another interesting empirical extension would be to examine the cross-sectional relation between the quadratic risk premium and expected returns for other asset classes such as corporate bonds, currencies and commodities.

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Figure 1: S&P 500 Quadratic Payoff and Realized Variance (Daily Returns)

In Panels A and B of this figure, we plot the time-series of the S&P 500 realized autocovariance (RA) and standardized realized autocovariance, respectively. In Panel C, we plot the quadratic loss (QL) and loss realized variance (RV), while in Panel D we plot the quadratic gain (QG) and the gain RV. Realized autocovariance and standardized realized autocovariance are defined as following:

$$RA = rac{r^2 - RV}{2}, \; Std \; RA = rac{r^2 - RV}{r^2 + RV}$$

where r^2 is the quadratic payoff, and RV is the realized variance. We obtain the expression for RA by solving for it in Equation (6). Standardized realized covariance multiplied by 100 yields the percentage of equity uncertainty represented by RA. Realized autocovariance, and all measures of the quadratic payoff and realized variance are in monthly squared percentage terms. The sample period is from January 1996 to December 2015.



Figure 2: Market and Firm Median Loss and Gain QRP

We plot the cross-sectional median firms' loss and gain quadratic risk premium (QRP), and the time series of the market loss and gain QRP. The left hand y-axis shows the values of the firm QRP, while the right hand y-axis shows the values for the market QRP. QRP are in monthly squared percentage terms. The sample period is from January 1996 to December 2015.





Table 1: Descriptive Statistics and Cross-Sectional Correlations

In Panel A we report the time-series mean, min, max, standard deviation (StdDev), skewness, excess kurtosis, and persistence (AR(1)) of the firm-level median and market quadratic risk premium (QRP^l , QRP^g , QRP) and risk-neutral skewness (SKEW). We also report these statistics for the firm-level median stock illiquidity (ILLIQ), idiosyncratic volatility (IVOL), book-to-market (B/M), past 12-month cumulative excess return (P12M), and one-month cumulative excess return (P01M). All statistics are monthly values. The mean, min, max, and standard deviation of QRP are in percentage-square units. The mean, min, max, and standard deviation of QRP are in percentage-square units. The mean, min, max, and standard deviation of QRP are in percentage-square units. The mean, min, max, and standard deviation of QRP are in percentage-square units. The mean, min, max, and standard deviation of QRP are in percentage-square units. The mean, min, max, and standard deviation of QRP are in percentage-square units. The mean, min, max, and standard deviation of QRP are in percentage-square units. The mean, min, max, and standard deviation of QRP are in percentage-square units. The mean, min, max, and standard deviation of QRP are in percentage-square units. The mean, min, max, and standard deviation of QRP are in percentage-square units. The mean, min, max, and standard deviation of QRP are in percentage-square units. The mean, min, max, and standard deviation of QRP are in percentage-square units. The mean, min, max, and standard deviation of QRP are in percentage-square units. The mean, min, max, and standard deviation of QRP are in percentage-square units. The mean, min, max, and standard deviation of QRP are in percentage of the correlations in two steps. First, in each month t we compute cross-sectional correlations among all variables. This yields a monthly time-series of cross-sectional correlations. Next, we take the time-series average of these correlations, and these are the correl

Panel A: Descriptive Statistics

	Mean	Min	Max	StdDev	Skewness	Kurtosis	AR(1)
QRP^{l}	32.57	-0.70	211.35	33.82	2.41	10.60	0.83
QRP^{g}	13.85	-20.78	69.59	10.83	1.46	9.00	0.66
QRP	13.82	-23.83	204.80	30.29	2.66	13.73	0.67
ILLIQ	4.8e-3	7.9e-4	0.02	1.1e-4	0.90	2.81	0.95
SKEW	-0.51	-1.31	0.21	0.22	-0.06	2.50	0.73
IVOL	2.04	1.13	4.16	0.70	0.99	3.20	0.91
B/M	0.45	0.30	1.03	0.09	1.82	8.05	0.93
P12M	6.19	-49.50	68.52	19.45	-0.02	3.80	0.92
P01M	0.32	-22.52	16.78	5.30	-0.69	5.06	0.13
QRP_m^l	15.65	-3.66	80.02	14.32	1.65	6.22	0.79
QRP_m^g	2.92	-14.52	47.74	7.11	1.82	12.58	0.49
QRP_m	12.73	-17.48	79.17	16.84	1.16	4.53	0.66
$SKEW_m$	-1.96	-3.79	-0.73	0.59	-0.48	2.88	0.86

Panel B: Cross-sectional Correlati

	QRP^{g}	QRP	ILLIQ	SKEW	IVOL	B/M	Size	P12M	P01M
QRP^{l}	-0.20	0.87	0.11	-0.06	0.14	0.03	-0.04	5.4e-4	0.05
QRP^{g}		-0.61	-0.01	-0.03	0.15	-0.02	-0.04	0.08	0.09
QRP			0.09	-0.04	0.03	0.03	-0.01	-0.03	1.6e-3
ILLIQ				0.10	0.20	0.09	-0.04	-0.09	-0.02
SKEW					0.15	-0.01	-0.17	0.03	-0.16
IVOL						0.06	-0.15	-0.08	0.11
B/M							-0.05	-0.18	-0.08
Size								0.02	0.01
P12M									0.01

Table 2: Univariate Sorts

In Panel A, at the end of month t we sort firms into quintiles based on their average loss QRP (QRP^l) during month t, so that Quintile 1 contains the stocks with the lowest QRP^l and Quintile 5 the highest. We then form value-weighted portfolios of these firms, holding the ranking constant for the next month. Subsequently, we compute cumulative returns during month t+1for each quintile portfolio. We report the monthly average cumulative return in percentage of each portfolio. Similarly, in Panel B and C, we sort firms into quintiles based on their average gain QRP (QRP^g) and net QRP (QRP), respectively. We also compute the Jensen alpha of each quintile portfolio with respect to the Fama-French five-factor model (Fama and French, 2015) by running a time-series regression of the monthly portfolio returns on monthly MKT, SMB, HML, RMW, and CMA. The t-statistics test the null hypothesis that the average monthly cumulative return of each respective portfolio equals zero, and they are computed using Newey and West (1987) standard errors to account for autocorrelation, and are reported in parentheses. Significant t-statistics at the 95% confidence level are boldfaced. QRP is reported in monthly square percentage units. Data are from January 1996 to December 2015.

		Panel A	: Firm Lo	oss QRP				Panel B: Firm Gain QRP					
			Quintiles				-		(Quintiles			
	1	2	3	4	5	5-1		1	2	3	4	5	5-1
QRP^l	-145.96	8.54	33.00	67.42	231.63		QRP^{g}	-59.68	-2.66	14.07	38.42	163.99	
$\mathbb{E}\left[r\right]$	-0.98 (- 2.15)	0.29 (0.98)	0.98 (2.97)	1.35 (3.12)	2.10 (3.92)	3.08 (7.79)		-0.97 (- 2.31)	0.17 (0.54)	0.84 (2.77)	0.90 (2.28)	1.98 (3.80)	2.95 (8.51)
alpha	-1.59 (- 7.47)	-0.19 (-1.72)	0.43 (3.75)	0.65 (4.02)	1.20 (4.59)	2.79 (6.82)		-1.59 (- 8.49)	-0.34 (- 3.11)	0.30 (3.26)	0.25 (2.01)	1.19 (5.14)	2.78 (7.94)

			Quintiles			
	1	2	3	4	5	5-1
QRP	-240.41	-21.57	14.07	51.88	236.54	
$\mathbb{E}\left[r\right]$	0.10 (0.19)	0.57 (1.79)	0.59 (2.03)	0.71 (1.94)	0.66 (1.45)	0.56 (1.74)
alpha	-0.61 (- 3.05)	0.05 (0.44)	0.11 (1.66)	0.08 (0.60)	-0.15 (-0.72)	0.46 (1.33)

Panel C: Firm Net QRP

Table 3: Conditional Double Sorts on Systematic Risk

Stocks are first sorted in quintiles based on their exposure to systematic risk factors including: Farago and Tédongap (2018) five GDA factors (market factor, the market downside factor, the downstate factor, the volatility factor and the volatility downside factor), market loss and gain quadratic risk premium (Bollerslev, Tauchen, and Zhou, 2009), and market risk-neutral skewness (Chang, Christoffersen, and Jacobs, 2013). Next, stocks within each quintile of the given systematic risk factor exposure are further sorted in quintiles based on their loss quadratic risk premium in Panel A, and gain quadratic risk premium in Panel B. The table reports the difference in average excess returns between the top and the bottom quintile ($\mathbb{E}[r]$) based on loss and gain QRP and the Jensen alphas with respect to the Fama-French five-factor model (Fama and French, 2015). *t*-statistics based on standard errors computed using the Newey and West (1987) procedure are reported in parentheses. Significant *t*-statistics at the 95% confidence level are boldfaced. Data are from January 1996 to December 2015.

		Panel A:	Loss QR	Р				Panel B:	Gain QR	Р	
			Quintiles	š					Quintiles		
	1	2	3	4	5		1	2	3	4	5
		M	arket Fac	tor				M	arket Fac	tor	
$\mathbb{E}\left[r\right]$	2.45	2.06	2.49	3.48	4.40	$\mathbb{E}\left[r ight]$	2.57	2.20	2.73	3.13	3.73
	(4.80)	(5.07)	(5.05)	(7.11)	(7.12)		(5.36)	(5.26)	(7.95)	(7.15)	(5.92)
alpha	2.47	2.08	2.44	3.19	4.37	alpha	2.59	2.02	2.51	2.83	3.57
	(4.40)	(5.13)	(4.09)	(6.15)	(5.90)		(3.94)	(3.79)	(7.28)	(6.58)	(4.23)
		Market	Downsid	e Factor				Market	Downsid	e Factor	
$\mathbb{E}\left[r\right]$	4.08	3.01	2.71	2.14	3.72	$\mathbb{E}\left[r ight]$	3.56	2.34	2.94	2.95	4.54
	(6.32)	(5.41)	(7.02)	(4.62)	(6.16)		(5.71)	(5.35)	(7.41)	(5.45)	(8.81)
alpha	4.11	2.99	2.42	2.04	3.80	alpha	3.42	2.37	2.67	2.83	4.36
	(4.51)	$({\bf 4.29})$	(6.06)	(4.91)	$({\bf 5.23})$		(4.12)	(4.85)	(5.75)	(3.66)	(6.28)
		Dow	nstate F	actor				Dow	nstate Fa	actor	
$\mathbb{E}\left[r\right]$	3.85	3.21	2.86	2.65	3.66	$\mathbb{E}\left[r ight]$	3.51	2.61	2.73	2.45	4.10
	(6.46)	(6.22)	(5.96)	(5.26)	(6.97)		(5.80)	(5.39)	(6.31)	(5.67)	(8.50)
alpha	3.72	3.32	2.58	2.86	3.60	alpha	3.28	2.58	2.49	2.29	3.99
	$({\bf 5.02})$	(4.88)	$({\bf 4.39})$	$({\bf 5.49})$	$({\bf 5.53})$		$({\bf 4.42})$	$({\bf 4.21})$	(4.85)	(4.00)	$({\bf 6.29})$
		Vol	atility Fa	ctor				Vol	atility Fa	ctor	
$\mathbb{E}\left[r\right]$	4.41	2.90	2.17	2.40	3.54	$\mathbb{E}\left[r ight]$	3.84	2.85	2.44	2.13	3.79
	(6.96)	(5.67)	(5.00)	(5.53)	(6.52)		(6.91)	(5.79)	(5.42)	(5.55)	(6.83)
alpha	4.34	3.09	2.00	2.49	3.60	alpha	3.67	2.57	2.34	2.00	3.53
	$({\bf 5.04})$	(4.89)	$({\bf 4.36})$	(5.05)	$({\bf 5.46})$		$({\bf 5.31})$	(3.71)	(4.11)	$({\bf 5.31})$	(4.25)
		Volatility	y Downsi	de Factor				Volatility	y Downsie	de Factor	
$\mathbb{E}\left[r\right]$	3.28	3.04	2.21	3.14	3.80	$\mathbb{E}\left[r ight]$	3.41	2.93	1.96	2.27	4.32
	(5.75)	(5.45)	(4.82)	(5.90)	(6.35)		(5.74)	(6.39)	(4.82)	(4.82)	(9.02)
alpha	3.43	3.11	2.19	2.94	3.73	alpha	3.45	2.76	1.81	1.85	4.08
	$({\bf 4.57})$	$({\bf 4.72})$	(3.80)	(4.86)	$({\bf 4.92})$		(3.87)	$({\bf 4.72})$	$({\bf 4.12})$	(3.56)	(6.61)
		Mar	ket Loss	QRP				Mar	ket Gain	QRP	
$\mathbb{E}\left[r\right]$	3.27	2.79	2.87	2.62	4.60	$\mathbb{E}\left[r ight]$	3.26	2.68	1.96	2.59	4.00
	(5.82)	(5.56)	(6.00)	(5.34)	(7.88)		(6.31)	(8.05)	(4.98)	(5.27)	(7.24)
alpha	3.42	2.72	2.64	2.64	4.61	alpha	4.00	2.00	1.62	2.76	3.41
	(4.71)	(4.17)	(5.08)	$({\bf 5.55})$	$({\bf 5.52})$		(7.29)	$({\bf 4.49})$	(3.64)	(4.77)	$(\boldsymbol{6.09})$
	N	farket Ris	sk-Neutra	al Skewne	ess		N	farket Ris	sk-Neutra	d Skewne	ess
$\mathbb{E}[r]$	3.47	2.37	2.12	3.07	4.60	$\mathbb{E}\left[r\right]$	4.02	2.12	2.34	2.77	4.45
	(5.90)	(5.10)	(4.66)	(6.33)	(7.09)		(5.91)	(4.69)	(5.38)	(6.77)	(8.15)
alpha	3.14	2.40	2.34	2.87	4.55	alpha	3.75	1.92	2.03	2.60	4.22
-	(4.30)	(4.03)	(3.74)	(4.87)	(5.25)		(3.94)	(3.90)	(4.50)	(4.63)	(6.59)
	. /	. /	. /	. /	. /		· · /		. ,	. /	

Table 4: Conditional Double Sorts on Firm Characteristics

Stocks are first sorted in quintiles based on different characteristics: implied volatility smirk (Xing, Zhang, and Zhao, 2010; Yan, 2011), risk-neutral skewness (Bakshi, Kapadia, and Madan, 2003), relative signed jump variation (Bollerslev, Li, and Zhao, forthcoming), idiosyncratic volatility (Ang, Hodrick, Xing, and Zhang, 2006), illiquidity (Amihud, 2002), number of analysts covering the stock (Hong, Lim, and Stein, 2000), and a proxy for lottery demand (Bali, Cakici, and Whitelaw, 2011), respectively. Next, stocks within each characteristic quintile are sorted in quintiles based on loss QRP (Panel A), and gain QRP (Panel B). The table reports the difference in average excess returns between the top and the bottom quintile ($\mathbb{E}[r]$) based on loss or gain QRP and the Jensen alphas with respect to the Fama-French five-factor model (Fama and French, 2015). *t*-statistics based on standard errors computed using the Newey and West (1987) procedure are reported in parentheses. Significant t-statistics at the 95% confidence level are boldfaced. Data are from January 1996 to December 2015.

		Panel A:	Loss QR	Р				Panel B:	Gain QI	RP	
			Quintiles	;					Quintile	s	
	1	2	3	4	5		1	2	3	4	5
		Implied	l volatilit	y smirk				Implie	d volatili	ty smirk	
$\mathbb{E}[r]$	4.02	4.01	2.94	2.87	2.45	$\mathbb{E}\left[r ight]$	4.27	3.25	3.19	2.54	2.40
	(7.15)	(7.38)	(5.93)	(4.52)	(4.34)		(8.86)	(6.87)	(6.45)	(6.01)	(4.93)
alpha	3.81	3.70	2.94	2.93	2.51	alpha	4.30	2.85	3.03	2.49	2.15
	(5.48)	(5.28)	(5.26)	(3.63)	(4.06)		(7.47)	(5.40)	(5.50)	(4.63)	(3.33)
	I	Firm Risl	k-Neutral	Skewnes	s			Firm Ris	k-Neutra	l Skewne	ss
$\mathbb{E}\left[r\right]$	2.14	2.70	3.44	4.55	4.59	$\mathbb{E}\left[r ight]$	1.75	2.57	3.08	3.78	4.49
	(4.04)	(4.75)	(5.14)	(5.73)	(6.00)		(3.33)	(5.93)	(4.66)	(6.60)	(6.85)
alpha	2.13	2.71	3.44	4.52	4.52	alpha	1.60	2.35	3.02	3.60	4.22
	(4.30)	(4.73)	(5.06)	(5.67)	(5.94)		(2.87)	(5.08)	(4.47)	(6.32)	(6.36)
	Re	elative Sig	gned Jum	p Variat	ion		R	elative Si	gned Jur	np Variat	ion
$\mathbb{E}\left[r\right]$	4.06	2.82	2.66	2.78	2.86	$\mathbb{E}\left[r ight]$	3.26	3.05	2.62	2.79	3.31
	(5.84)	(4.95)	(4.37)	(5.22)	(4.33)		(5.84)	(5.12)	(5.40)	(3.54)	(4.77)
alpha	3.96	2.64	2.31	2.77	2.85	alpha	2.78	3.12	2.39	2.88	3.10
	(5.34)	(4.31)	(4.05)	(5.17)	(4.47)		(4.94)	(5.29)	(5.13)	(3.60)	(4.56)
		Idiosyr	neratic Vo	olatility				Idiosy	ncratic V	olatility	
$\mathbb{E}\left[r\right]$	0.94	2.68	3.23	3.87	5.58	$\mathbb{E}\left[r ight]$	1.18	1.87	2.48	4.03	5.84
	(3.53)	(6.26)	(4.81)	(5.17)	(4.39)		(3.36)	(4.26)	(5.51)	(6.82)	(5.99)
alpha	0.97	2.86	3.13	3.97	5.13	alpha	1.05	1.55	2.09	3.92	5.87
	(3.35)	(6.04)	(4.48)	(5.06)	(4.27)		(3.01)	(3.12)	(4.69)	(6.56)	(6.08)
		Sto	ock illiqui	dity				Ste	ock illiqu	idity	
$\mathbb{E}[r]$	1.84	2.75	3.16	4.42	4.93	$\mathbb{E}[r]$	1.67	2.32	3.73	4.29	5.09
	(4.96)	(6.99)	(7.23)	(8.53)	(9.50)		(4.96)	(6.00)	(7.77)	(8.41)	(10.87)
alpha	1.89	2.60	2.99	4.35	4.98	alpha	1.67	2.26	3.81	4.09	4.88
	(3.85)	(4.66)	(5.57)	(6.02)	(6.96)		(4.17)	(4.28)	(5.70)	(5.74)	(8.00)
									1		
TP []	9.57	Ana	lysts' cov	erage	1.09	TT []	4.59	Ana	lysts' cov	verage	1.00
$\mathbb{E}[r]$	3.57	4.23	3.66	2.30	1.82	$\mathbb{E}[r]$	4.52	3.98	2.62	2.28	1.90
	(6.81)	(6.82)	(7.98)	(4.83)	(4.63)	1.1	(8.63)	(7.31)	(5.83)	(5.27)	(5.43)
alpha	3.50	4.17	3.54	2.14	1.89	alpha	4.44	4.02	2.25	2.03	1.89
	(6.37)	(4.89)	(5.46)	(3.21)	(3.74)		(5.80)	(6.05)	(3.97)	(4.00)	(4.78)
		Let	terv dom	and				Ιa	tterv den	and	
F [r]	1.64	2.13	3 70	4.64	5.43	IF [m]	1.23	2 10	3.99	3 01	5.19
	(5.31)	(5.96)	(7.01)	(7.37)	(7.23)	112 [7]	(3.78)	(5.79)	(6.30)	(7.83)	(8.31)
alnha	1.54	2.04	3.86	4 46	5.40	alnha	1 18	2.03	2.84	3.64	4 95
aipiia	(4 10)	(5.18)	(5.14)	(5.83)	(5.09)	alpiia	(2.97)	(4.30)	(4.80)	(5.45)	(5.87)
	(4.10)	(0.10)	(0.14)	(0.00)	(0.00)		(2.01)	(4.00)	(4.00)	(0.40)	(0.01)

Table 5: Triple Sorts on SKEW and Idiosyncratic Volatility

In Panel A, stocks are sorted in terciles based on their loss QRP. Next, stocks within each tercile of loss QRP are further sorted in terciles based on their gain QRP. Finally, within each tercile of gain QRP stocks are sorted in terciles based on SKEW (Xing, Zhang, and Zhao, 2010; Yan, 2011). In Panel B, stocks are independently sorted every month in terciles based on their gain quadratic risk premium (QRP), loss QRP and idiosyncratic volatility (Ang, Hodrick, Xing, and Zhang, 2006), respectively. Next, we take the intersection of these tercile portfolios. We report Jensen alphas with respect to the Fama-French five-factor model (Fama and French, 2015) for all tercile portfolios as well as for the difference between the top and bottom tercile (H–L). t-statistics are computed using Newey and West (1987) standard errors, and are reported in parentheses. Significant t-statistics at the 95% confidence level are boldfaced. The sample period is from January 1996 to December 2015.

							Loss QRI)					
			L			М					Н		
			Gain QRI)		Gain QRP			Gain QRP				
		L	М	Н		L	М	Н		L	Μ	Н	
SKEW	L M H	-3.84 -3.77 -4.10	-1.00 -1.19 -1.37	-1.34 -0.79 -2.12	L M H	-0.92 -1.08 -0.99	-0.13 -0.01 -0.23	1.03 1.14 0.46	L M H	-0.23 -0.79 -1.29	$1.26 \\ 0.61 \\ 0.25$	3.99 3.09 2.46	
	H–L	-0.26 (-0.67)	-0.37 (-1.80)	-0.78 (-1.93)	H–L	-0.06 (-0.21)	-0.10 (-0.41)	-0.57 (-1.63)	H–L	-1.06 (- 2.19)	-1.01 (- 2.45)	-1.52 (- 2.69)	

Panel A: Conditional Triple Sorts on Loss QRP, Gain QRP, and SKEW

Panel B: Unconditional Triple Sorts on Loss QRP, Gain QRP, and IVOL

]	Loss QRI)					
			L			Μ			Н			
			Gain QRI	þ	(Gain QRI)		Gain QRP			
		L	М	Н	L	Μ	Н	L	М	Н		
IOOI	L M H H–L	-2.74 -4.07 -6.04 -3.31	-0.93 -1.73 -2.90 -1.97	-0.44 -1.01 -1.84 -1.40	-0.91 -0.89 -1.54 -0.63	-0.18 -0.16 -0.60 -0.42	1.09 0.65 0.65 -0.44	-0.57 -1.00 -1.57 -1.00	1.09 0.65 0.65 -0.44	2.49 2.84 2.86 0.45		
		(- 7.43)	(- 5.57)	(- 5.05)	(-1.34)	(-1.02)	(-0.89)	(-1.86)	(-0.89)	(0.78)		

Table 6: Fama-MacBeth Regressions Controlling for Systematic Risk

This table reports the time-series average of the monthly estimated coefficients for different factor models including firm quadratic risk premium (QRP^l, QRP^g) and QRP). In each regression from III to VII we include the firm loss and gain quadratic risk premium with different factor models: CAPM, market skewness factor model (Chang, Christoffersen, and Jacobs, 2013), market quadratic risk premium model (Bollerslev, Tauchen, and Zhou, 2009), Carhart four-factor model, and the GDA five-factor model (Farago and Tédongap, 2018), respectively. All coefficients are estimated using the Fama and MacBeth (1973) two-step regression applied on 5150 individual firms. In the first step, we regress six months of daily excess returns of the 5150 firms on the different factor models to obtain their respective betas. In the second step, we run cross-sectional regressions of month t + 1 firm excess returns against the estimated betas and firm quadratic risk premium. t-statistics are computed using Newey and West (1987) standard errors, and are reported in parentheses. Significant t-statistics at the 95% confidence level are boldfaced. Adjusted R^2 is reported in percentage. Data are from January 1996 to December 2015.

I		II III		IV		V		VI		VII			
Cst	0.01 (0.08)	Cst	-7.5e-4 (- 3.24)	Cst	0.01 (0.57)	Cst	0.01 (0.58)	Cst	0.01 (0.69)	Cst	0.01 (0.34)	Cst	0.01 (0.78)
QRP	0.07 (1.60)	QRP^{l}	0.616 (8.90)	QRP^{l}	(0.609) (11.24)	QRP^{l}	0.610 (11.28)	QRP^{l}	(0.62) (12.70)	QRP^l	0.610 (14.10)	QRP^l	0.624 (12.13)
		QRP^{g}	1.348 (12.06)	QRP^{g}	1.444 (14.97)	QRP^{g}	1.440 (14.79)	QRP^{g}	1.461 (15.72)	QRP^{g}	1.475 (18.30)	QRP^{g}	1.467 (16.16)
				$\beta_{m,CAPM}$	-0.01 (-0.88)	$\beta_{m,SKEW}$	-0.01 (-0.08)	$\beta_{m,BTZ}$	-0.01 (-0.43)	$\beta_{m,CH}$	-0.01 (-0.28)	$\beta_{m,W}$	-0.01 (-0.03)
						β_{MSKEW}	$0.09 \\ (1.03)$	β_{MQRP^l}	-1.9e-6 (-0.45)	β_{smb}	-2.9e-3 (- 3.37)	β_X	1.4e-5 (3.29)
								β_{MQRP^g}	2.9e-6 (2.11)	β_{hml}	-7.2e-4 (-0.55)	β_{D}	$0.26 \\ (5.02)$
								β_{VIX}	8.4e-6 (1.49)	β_{mom}	2.3e-3 (1.97)	β_{WD}	-0.01 (- 3.13)
												β_{XD}	1.3e-5 (1.83)
Adj. R^2	1.22		4.22		7.88		8.26		9.21		11.91		9.45

Table 7: Fama-MacBeth Regressions Controlling for Other Firm Characteristics

This table reports the time-series average of the monthly estimated coefficients for factor models including firm quadratic risk premium $(QRP^l, QRP^g \text{ and } QRP)$. In regression VIII we include the firm loss and gain quadratic risk premium with the relative signed jump variation (RSJ) from Bollerslev, Li, and Zhao (forthcoming). In regression IX we include the firm loss and gain quadratic risk premium with all the firm characteristics: RSJ, idiosyncratic volatility (IVOL) computed as in Ang, Hodrick, Xing, and Zhang (2006), past 1-month cumulative excess return (P01M), past 12-month cumulative excess return (P12M), size, book-to-market (B/M), illiquidity (ILLIQ), risk-neutral skewness (FSKEW), the loss and gain realized semi-variances $(RV^l \text{ and } RV^g)$, and firm risk neutral skewness. All coefficients are estimated using the Fama and MacBeth (1973) two-step regression applied on 5150 individual firms. We run cross-sectional regressions of month t + 1 firm excess returns against firm characteristics and firm quadratic risk premium. t-statistics are computed using Newey and West (1987) standard errors, and are reported in parentheses. Significant t-statistics at the 95% confidence level are boldfaced. Adjusted R^2 is reported in percentage. Data are from January 1996 to December 2015.

	Ι]	I	V	III	IJ	IX		
Cst	0.01 (0.08)	Cst	-7.5e-4 (- 3.24)	Cst	-7.7e-4 (- 3.79)	Cst	-2.3e-3 (-0.76)		
QRP	0.07	QRP^{l}	0.616	QRP^{l}	0.625	QRP^{l}	0.848		
	(1.60)		(8.90)		(8.75)		(13.37)		
		QRP^{g}	1.348	QRP^{g}	1.359	QRP^{g}	1.677		
			(12.06)		(12.55)		(13.02)		
				RSJ	-0.01	RSJ	4.0e-4		
					(-2.62)		0.29		
						IVOL	-0.20		
						DOIM	(-2.88)		
						P01M	-0.02		
						$\mathbf{P}19\mathbf{M}$	(-2.90)		
						1 12101	(0.62)		
						Size	(0.02) 3 5e-4		
						Size	(0.61)		
						B/M	0.01		
							(2.32)		
						ILLIQ	-0.38		
						-	(-1.04)		
						RV^l	-0.11		
							(-3.05)		
						RV^g	-0.17		
							(- 3.06)		
						FSKEW	0.01		
							(6.25)		
Adj. R^2	1.22	Adj. R^2	4.22	Adj. R^2	5.09	$\mathrm{Adj.}R^2$	11.65		